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Vol. VI NOVEMBER, 1922 Number 9

NOTE ON THE ENERGY EXCHANGES IN THE FORMATION OF THE LATENT IMAGE OF A PHOTOGRAPHIC EMULSION[1]

By S. E. Sheppard and E. P. Wightman

A number of determinations have been made of the energy of light of different wave-lengths necessary to produce, on development, a blackening of unit density of various photographic plates; and also the energy necessary to produce the least perceptible visible blackening has been found for many commercial plates.

Not so many calculations have been made, however, of the energies involved in the formation of the latent image itself. Mees,[2] in 1915, gave certain calculations of Nutting as to the amount of energy necessary, after full development, to produce a deposit of unit density and from this deduced that "the energy incident on a grain during exposure may be sufficient to affect only one molecule in the grain, and the latent image," he said, "may be composed of grains in each of which, on the average, only one molecule has lost an electron by the action of light."

Lately, Volmer[3] made some calculations, based on the measurements of Leimbach,[4] of the ratio of altered to unaltered moles of

[1] Communication No. 151 from the Research Laboratory of the Eastman Kodak Company.

[2] C. E. K. Mees, J. Franklin Inst., *179*, p. 164; 1915. A recent recalculation of Nutting's value shows that it is probably much too small.

[3] M. Volmer, Phot. Korr. *58*, p. 226; 1921.

[4] G. Leimbach, Z. wiss. Phot. *7*, 181; 1909. A preliminary determination in this laboratory of a similar value gives an amount of energy of the same order of magnitude.

silver bromide in a grain of a certain size. He used for this purpose, however, the energy of blue light, λ = about 450$\mu\mu$, which would produce a density of unity, and failed to take into account that only about ½ the total surface of the grain would be exposed. He assumed the grain to be spherical, whereas in most emulsions, grains of the size which he used for his calculations are tabular with an average thickness from ½ to 1/12 the mean width.

Instead of the energy used by Volmer for his calculations let us take Leimbach's value for the "Schwellenwert," that is, the visible threshold value after development for the Schleussner plate—upon which the former made his calculations.

For $\lambda = 450\mu\mu$ this is given as

$$\epsilon_1 = 10.5\times10^{-11}\ \frac{\text{watt}}{\text{cm}^2}\ \text{sec.}$$

which in ergs is

$$= 10.5\times10^{-3}\ \text{ergs/cm}^2$$

Assume, for convenience, that the grain has the shape of a square tablet with length of side $=10^{-4}$ cm, and thickness $=10^{-5}$ cm (this is much nearer the average size grain in photographic emulsions than that assumed by Volmer). The energy per quantum is

$$\epsilon_2 = h\nu = \frac{hc}{\lambda} = \frac{6.6\times10^{-27}\times3\times10^{-10}}{4.5\times10^{-5}} = 4.37\times10^{-12}\ \text{ergs.}$$

Then from Einstein's photochemical law, the number of molecules affected by the light will be

$$n = \frac{\epsilon_1}{h\nu} = \frac{\epsilon_1}{\epsilon_2} = \frac{10.5\times10^{-11}}{4.37\times10^{-12}} = 24.0\ \text{molecules per grain.}$$

The total number of molecules per grain may be calculated in either of two ways, from its weight or from the lattice structure. Both give the same result. Take the first case and assume the above values for width and thickness and the density of silver bromide crystals to be 6.47. The volume is then

$$v = 10^{-4}\times10^{-4}\times10^{-5} = 10^{-13}\ \text{cc.}$$

and its weight is,

$$M = 6.47\times10^{-13}\ \text{g}$$

Dividing by the weight of one molecule of AgBr, $m = 1.66 \times 10^{-24} \times 188 = 3.12 \times 10^{-22}$, we get for the number of molecules of AgBr in the grain

$$N = 2.07 \times 10^{9}$$

On the other hand if we calculate from the lattice structure and use Wilsey's value[5] of the lattice constant, 2.89 A. U. $= 2.89 \times 10^{-8}$ cm, then

$$N = \frac{10^{-4}}{2 \times 2.89 \times 10^{-8}} \cdot \frac{10^{-4}}{2.89 \times 10^{-8}} \cdot \frac{10^{-5}}{2.89 \times 10^{-8}} = 2.06 \times 10^{9}$$ molecules per grain

The number of molecules in the surface on which the light falls is of course obtained by omitting the last factor, the thickness, in the preceding equation, and is

$$N_s = 6 \times 10^{8}$$ molecules per surface exposed

The ratio of altered to unaltered molecules is therefore

$$n : N = 1 : 8.6 \times 10^{8}$$

and

$$n : N_s = 1 : 2.5 \times 10^{5}$$

This ratio is different, of course, for every different sized grain but for this particular grain it is still too large,[6] that is, if we consider, not the visible threshold value, but the actual threshold value, which could only be detected by the high power microscope.

As a matter of fact, work in the last few years on crystal structure in general, and on the crystal structure of AgBr in particular, by means of X-ray analysis; and on the applications of the quantum theory of light, have changed our views of the mechanism of the formation of the latent image.

It is certain that we cannot regard the grain of AgBr any longer as a heterogeneous clumping of molecules, and hence in speaking of "molecules per grain" all we mean is the number of pairs of (Ag^+, Br^-) that occur in the grain.

[5] R. B. Wilsey, Phil. Mag., *13*, p. 262; 1921.

[6] If just 1 molecule is affected by 1 quantum of energy then $n = \frac{\epsilon_1}{\epsilon_2} = 1$, and the ratio for the exposed surface becomes $n : N_s = 1 : 6 \times 10^{8}$. It might be of interest in this connection to note that R. Gans (Ann. d. Physik. *52*, p. 291; 1917) has detected quantities of light smaller than the quantum, but it is not known if these would be sufficient to affect a photographic plate.

The mechanism itself of latent image formation, we shall not discuss here but would refer to two papers[7] by one of us recently published.

Let us consider the possible energy changes which can occur when light falls on the grain:

1. *Energy of separation of Ag^+ from Br^- in the lattice structure:*
$$(Ag^+, Br^-) = Ag^+ + Br^- - \Delta_1 \text{ cal.} \quad (1)$$
2. *Energy of the loss of an electron by the Br^- ion:*
$$Br^- - \Theta = Br \text{ (liquid)} + \Delta_2 \text{ cal.} \quad (2)$$
3. *Energy of the addition of an electron to the Ag^+ ion:*
$$Ag^+ + \Theta = Ag \text{ (atom, solid state)} - \Delta_3 \text{ cal.} \quad (3)$$
4. *Energy of formation of Br_2 molecule:*
$$2\,Br = Br_2(gas) + \Delta_4 \text{ cal.} \quad (4)$$
5. *Energy of formation of Br addition product with gelatin:*
$$\text{Gelatin} + Br = (\text{Gelatin. } Br) + \Delta_5 \text{ cal.} \quad (5)$$
6. *Energy of formation of Br substitution product with gelatin:*
$$\text{Gelatin} + 2\,Br = (R\ .\ Br) + HBr - \Delta_6 \text{ cal.} \quad (6)$$

NOTE: The gelatin−Br substitution product would probably be an endothermic compound but the formation of HBr is exothermic to a greater extent, hence Δ_6 cal. is positive.

7. *Energy of loss of an electron by the silver molecule or atom:*
$$Ag - \Theta = Ag^+ + \Delta_7 \text{ cal.} \quad (7)$$

Of these seven possibilities the first five appear to be the most probable; the seventh may play an important catalytic part, however, in that silver metallic silver atoms may supply the electron for reversing equations (3) and (2), and assist the local surface concentration of the latent image. Under such conditions, therefore, the previous energy changes become of less relative importance. We shall return to the evaluation of these energy changes in a later paper.

EASTMAN KODAK COMPANY,
ROCHESTER, N. Y.,
JULY 31, 1922.

[7] S. E. Sheppard, Silver nucleus theory of development, Phot. Korr, Jan.-April, 1922, p. 76. The action of soluble iodides and cyanides on the photographic emulsion, Phot. J. *60*, p. 88; 1922.

THE OPTICAL CONSTANTS OF ISOLATED TELLURIUM CRYSTALS

By George Dewey Van Dyke

This paper is a brief report on one phase of an extended research which is being conducted by Dr. L. P. Sieg of the State University of Iowa, on the optical properties of small metallic crystals. The information obtained from such investigations should reveal much in regard to the crystalline structure of the pure metal and possibly throw some light on the atomic structure of metals in general.

Skinner[1] has published results on the optical constants of a selenium crystal, finding the crystal doubly refracting, the index when the principal axis of the crystal is parallel to the plane of incidence being higher than that of any other known element. Skinner employed the ordinary polarimetric method of determining the elliptic constants. Weld[2] has made a fairly extensive study of the elliptic constants of light reflected from a small selenium crystal using the "Crystelliptometer," a special apparatus designed by himself. The same apparatus was used by the writer in obtaining the optical constants of tellurium crystals. The crystals were made by sublimation at the State University of Iowa, by Dr. A. R. Fortsch.

Drude[3] has developed the theory of metallic reflection for crystalline bodies.

Let Φ_1, Δ_1, Ψ_1, be respectively, the angle of incidence, phase difference, and azimuth when the principal crystal axis is parallel to the plane of incidence;

Φ_2, Δ_2, Ψ_2 be, respectively, the angle of incidence, phase difference, and azimuth when the principal axis of the crystal is perpendicular to the plane of incidence, and

n_1, n_2, k_1, k_2, ρ_1, ρ_2, be the indices of refraction, absorption coefficients, and coefficients of reflection, respectively, for the

[1] Skinner, Phys. Rev., N. S., *9*, p. 148; 1917.

[2] Weld, *Journal of the Optical Society of America*, *6*, p. 67; 1922.

[3] Drude, Ann. d. Physik, *34*, p. 529; *32*, p. 616, 1887; *35*, p. 518; 1888.

two principal positions of the crystal axis, n_1, referring, for example, to the index of refraction when the electric vector agrees with the principal axis.

Then

$$\frac{\sqrt{\alpha}}{\cos\phi_1}-\sqrt{\gamma}\cos\phi_1=\frac{\cos 2\Psi_1+i\sin 2\Psi_1\sin\Delta_1}{1-\sin 2\Psi_1\cos\Delta_1}$$

$$\frac{\sqrt{\gamma}}{\cos\phi_2}-\sqrt{\alpha}\cos\phi_2=\frac{\cos 2\Psi_2+i\sin 2\Psi_2\sin\Delta_2}{1-\sin 2\Psi_2\cos\Delta_2}$$

where α and λ are auxiliary complex constants employed in determining the final optical constants. The letter i represents $\sqrt{-1}$.

Let the values of α and γ as given by the above equations be

$$\alpha=a_{11}+i\,a_{12} \qquad\qquad \gamma=a_{21}+i\,a_{22}$$

Then if we define X by $\tan X=\frac{a_{12}}{a_{11}}$,

and E by $\tan E=\frac{a_{22}}{a_{21}}$, we have

$$k_1=\tan\frac{X}{2}, \qquad\qquad k_2=\tan\frac{E}{2},$$

$$n_1^2=\frac{2\sin\frac{X}{2}\cos^3\frac{X}{2}}{a_{12}}, \qquad\qquad n_2^2=\frac{2\sin\frac{E}{2}\cos^3\frac{E}{2}}{a_{22}}$$

$$\rho_1=\frac{n_1^2(1+k_1^2)+1-2n_1}{n_1^2(1+k_1^2)+1+2n_1}, \text{ and} \qquad\qquad \rho_2=\frac{n_2^2(1+k_2^2)+1-2n_2}{n_2^2(1+k_2^2)+1+2n_2}$$

It should be noted that the values of α and γ do not depend upon the angles of incidence, hence for each wave-length we are able to determine α and γ for several angles of incidence and average the results.

The optical constants for five different wave-lengths in the visible spectrum were obtained. The results of the investigation (c. f. Figs. 1, 2, and 3) show that a crystal of tellurium is doubly refracting, the index of refraction being higher in the horizontal than in the vertical position. The reflecting power of the crystal

Table of Elliptic Constants

λ ($\mu\mu$)	Φ	Δ_1	$2\Psi_1$	Δ_2	$2\Psi_2$
437	60°	141°30′	74°	136°	66°48′
437	65°	126°6′	73°48′	119°24′	64°12′
437	70°	108°54′	72°12′	103°54′	60°36′
437	75°	89°42′	69°48′	78°54′	61°
450	60°	127°30′	69°12′	120°24′	63°24′
450	65°	117°12′	68°48′	113°36′	61°12′
450	70°	99°30′	66°24′	86°30′	62°48′
508	60°	136°30′	73°36′	133°24′	61°12′
508	65°	122°6′	67°24′	119°48′	58°24′
508	70°	108°36′	62°24′	105°	56°
508	75°	84°12′	61°48′	76°36′	55°12′
590	60°	137°36′	71°	133°	67°
590	65°	132°12′	65°12′	117°18′	60°24′
590	70°	108°6′	64°48′	100°36′	57°12′
590	75°	89°18′	59°24′	80°6′	54°24′
650	60°	127°54′	68°48′	115°12′	57°24′
650	65°	115°30′	65°	102°24′	55°48′
650	70°	102°24′	62°	92°42′	55°36′

Averages of Crystal Constants

λ	α	γ
437	$.0528+.0503i$	$.0513+.0811i$
450	$.0751+.0918i$	$.0800+.1232i$
508	$.0493+.0738i$	$.0391+.0967i$
590	$.0416+.0688i$	$.0426+.0896i$
650	$.0659+.1034i$	$.0641+.1532i$

Table of Optical Constants

	Crystal Axis Horizontal			Crystal Axis Vertical		
λ	n_1	κ_1	ρ_1	n_2	κ_2	ρ_2
437	3.44	.399	.33	2.52	.551	.30
450	2.62	.466	.28	2.29	.543	.26
508	2.96	.535	.31	2.57	.672	.30
590	3.07	.563	.34	2.68	.632	.30
650	2.50	.548	.29	2.05	.666	.27

varies very little for the two positions and for the various wave-lengths used, a fact verified by Sieg[4] by direct measurement of the reflecting power. The curve obtained from plotting the index of refraction against the wave-length shows a distinct

[4] Unpublished results.

minimum and maximum over the range investigated. This characteristic is also shown in Skinner's[5] results for selenium,

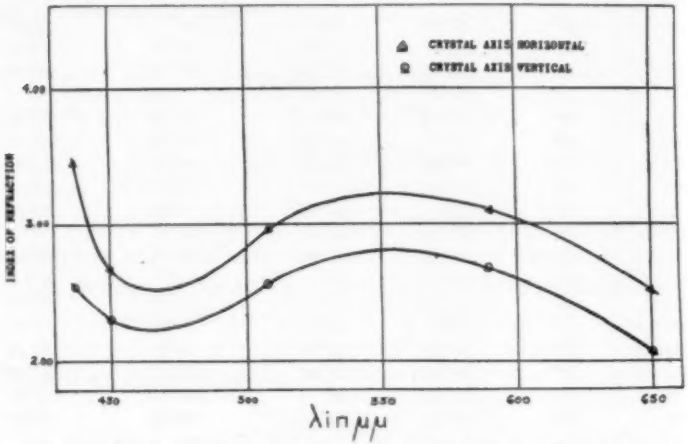


FIG. 1. *Principal indices of refraction of an isolated tellurium crystal for various wave-lengths.*

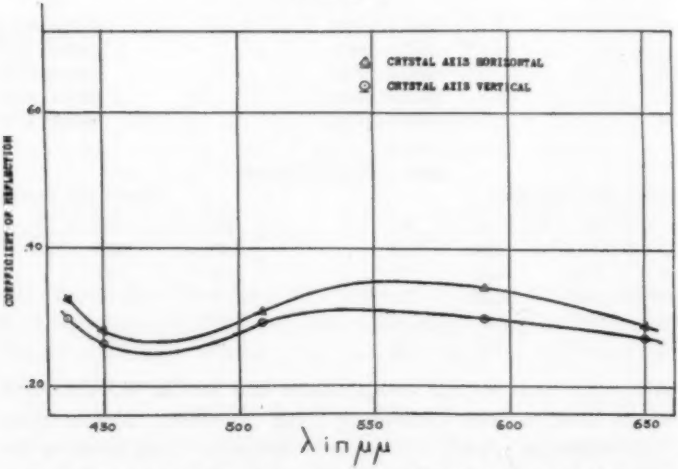


FIG. 2. *Principal reflecting powers of an isolated tellurium crystal for various wave-lengths.*

[5] Loc. cit.

although to a far less degree. The peculiarity found in the index in the horizontal position was not evident in tellurium, the two indices following almost similar curves.

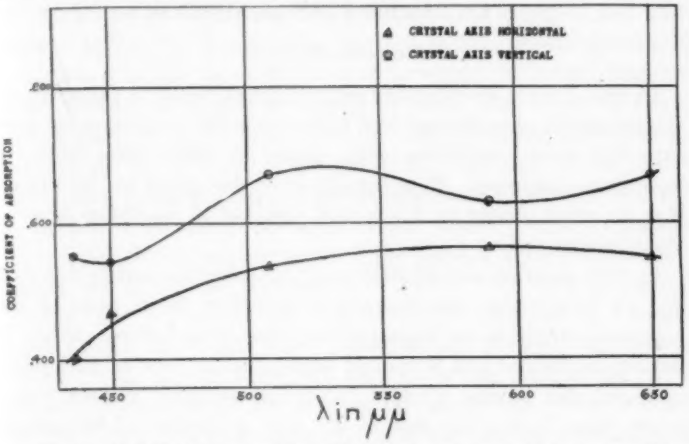


FIG. 3. *Variation in the coefficients of absorption of a crystal of tellurium with a variation in the wave-length.*

The above problem was suggested to me by Professor L. P. Sieg, to whom I wish to express my appreciation.

STATE UNIVERSITY OF IOWA,
AUGUST, 1922.

THE STUDY OF VISUAL PROCESSES BY MEANS OF MOMENTARY RETINAL SHADOWS

By Frederick W. Ellis

In 1901 I made an extensive series of experiments in the study of certain visual phenomena,[1] and I have recently supplemented and extended these researches with others in which new methods have been employed. It is the object of this paper to call attention to what seems to be a new method of studying several visual phenomena.

In 1891 Jastrow and Moorehouse[2] published an article describing an interesting phenomenon which had been brought to Jastrow's attention by Münsterberg some time before. When a slender horizontal rod is moved vertically in front of a rapidly revolving disc having a white or colored sector of greater luminosity than that of the rest of the disc, a number of horizontal bands are seen. Jastrow and Moorehouse studied this phenomenon under varying conditions, but gave no explanation of it, and spoke of it as an optical illusion. The word illusion does not apply to it, for it is due to well known optical and physiological laws. It demonstrates in a rather novel way the persistence and independence of visual perceptions, and affords a method of measuring the duration of the perceptions. The bands are due to the perception, and the persistence of the perception of the retinal shadow of the rod which is formed at each passage of the sector behind it. The word shadow is used here to denote an area of diminished luminosity irrespective of color.

In order to obtain a clear understanding of the mode of production of the bands it is best to use a white sector on a black ground. The sector should be comparatively narrow; one of twenty degrees is sufficiently broad. When the disc revolves rapidly enough to eliminate flicker it assumes a uniform dark gray shade. If a

[1] Studies in the Physiology and Psychology of Visual Sensations and Perceptions, American Journal of Physiology, Vol. 5.

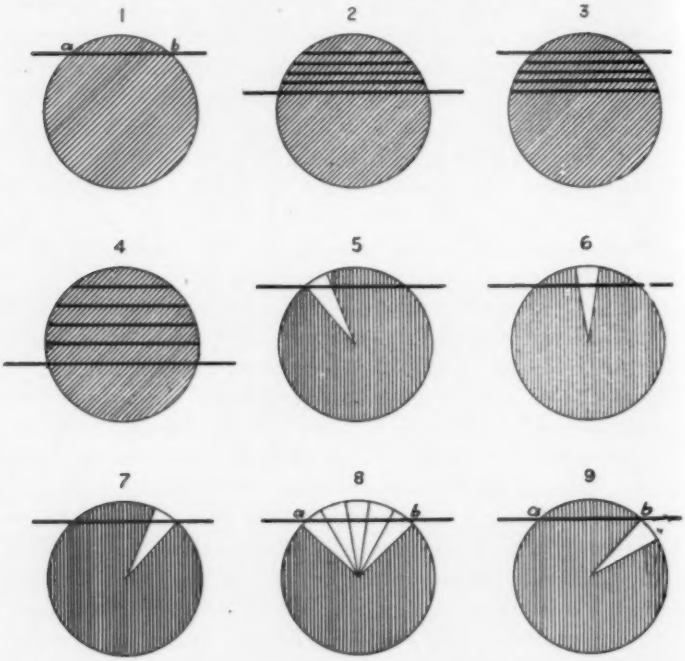
[2] American Journal of Psychology, Vol. 4.

slender black rod or strip, held horizontally, is now moved up and down at the rate of about once in a second, a number of very distinct black bands are seen on the dark gray background. If the motor to which the disc is attached is stopped, and then slowly turned by hand while the rod is held in its horizontal position, it is easy to study the way in which the retinal shadows are formed. It is only necessary to recall that what we see is the projection of the retinal image. As the white sector sweeps behind the rod a variable portion of it is covered by the rod at successive instants, and the retinal image or shadow of the rod is formed in successive sections. When the disc revolves rapidly the continuously varying parts of the shadow are perceived as a whole, owing to the persistence of vision. At each revolution of the disc the sector encounters the moving rod in another position in the field, and, consequently, another shadow is formed. As the persistence of vision is a considerable part of a second, several bands are seen at the same time. The number of the bands perceived depends upon the rate of revolution of the disc, and the distance between them is governed by the rate at which the rod is moved across the disc.

Figs. 1 to 9 illustrate the mode of formation of the bands. The oblique lines of the first four figures represent the fusion color or shade of the rapidly revolving disc; the vertical shading of the remaining figures indicates that the disc is at rest, or moving so slowly that it can be readily followed with the eye. The heavy horizontal line, extending beyond the disc on either side, corresponds to the horizontal rod, and the other broad lines, which are parallel to this line and terminate at the border of the disc, represent the shadows that we are studying. In Fig. 1 the rod is not moving; in Fig. 2 the rod is moving downward, and in Fig. 3 upward. Fig. 4 illustrates the appearance when the rod moves downward more rapidly than in Fig. 2. No sectors are seen in these four figures, for, in accordance with Talbot's law, the color of the sector is lost in the uniform fusion color of sector and ground. It is evident that none of these four figures represents a real retinal image. We may regard the four as psychic images due to the composite blending in the brain of

successive retinal images. The formation of these images is easily explained by referring to the remaining figures.

If the disc revolves clockwise, Fig. 5 will be an instantaneous view of it when the sector has just passed behind the rod. That part of the image of the rod which crosses the sector obliquely divides the sector into two parts separated by a retinal shadow,



or an area of less luminosity. Fig. 6 is a retinal image at a little later period, and the next figure one still later at the instant when the sector begins to emerge from behind the rod. In these instantaneous views it will be seen that the retinal shadows occupy varying positions with respect to the radial axis of the sector. Fig. 8 represents a composite of five different images in which it will be seen that the retinal shadows lie on the same straight line. Owing to the combination and persistence of the

changing impressions a shadow seems to extend from *a* to *b*. In Fig. 1 the projected shadow coincides with the stationary rod. If the rod is moved vertically downward while the disc completes its revolution the shadow will be seen behind the rod. When the disc makes the next revolution another subjective shadow is formed, and is seen directly behind the rod, while the first shadow still persists, and forms the second band. If the first shadow persists during five revolutions, the rod and bands will have the appearance shown in Fig. 2, and the band farthest from the rod will be the one first formed. It is evident from Fig. 9 that no retinal shadow will be formed while the sector continues its revolution from *b* back to *a*, and, if the rod moves during this period, it will give rise to another band when the sector again passes behind it.

The dark bands can also be shown very distinctly when a part of the revolving disc is a colored sector of considerable luminosity, and the ground is black. The distinctness of the bands varies with the brightness of the colored sector. When the colored disc is placed on a gray disc of the same luminosity the bands almost entirely disappear. Based upon this fact is a method of color photometry first employed by Rivers[3] in measuring the luminosity of colored papers by comparing them with a series of gray papers. He found that the bands nearly or completely disappeared when the gray had the same luminosity as the sector, and the results that he obtained in this way were nearly the same as those in a parallel series of experiments with the flicker method.

A complementary experiment with a slit in a sheet of black cardboard demonstrates in a striking way the mode of formation of the bands which we have been studying, and confirms my previous statements regarding their origin. The slit should be 4 or 5 mm wide, and of any convenient length. When the slit is held horizontally, in front of the revolving disc, and the screen is moved up and down about once in a second, a series of light bands corresponding to the dark ones of the previous experi-

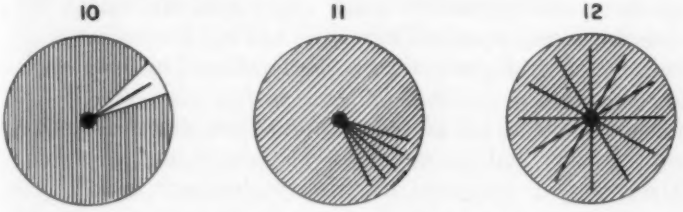
[3] The Journal of Physiology, *22*, 1897.

ments, and having a color resembling that of the sector, is seen. The colored sector as it passes behind the slit illuminates its different parts at successive instants, and these partial views are blended into an impression of the entire slit by the persistence of vision. The point to emphasize in this connection is that these images of the slit correspond to the dark bands of the rod experiment, and prove that the explanation of the origin of these bands given above is the correct one, and that they may be illustrated by the same figures.

It is possible to demonstrate another set of subjective shadows of different origin from that of those which we have described, if the revolving disc is placed in a darkened room, and is illuminated by a single lamp placed at a little distance in front of it. When the white sector is behind the rod, under these conditions, a real shadow of part of the rod is cast on the sector at each instant while it passes behind the rod. This comparatively short objective shadow sweeps across the disc in a nearly straight line, and gives rise to the subjective impression of a dark band extending across the disc, when the disc is regarded from one side. When the position of the observer is favorable two sets of bands may be seen; those due to retinal shadows, and those caused by the projection of a real shadow on the sector. It is evident that both kinds of bands may be similarly explained by the figures already given. The bands due to the retinal shadows of the rod are the more important, as they may be obtained under a greater variety of conditions, and are the ones best adapted for utilization in experimental work.

It is evident that the experiments which have been described afford the basis for two methods of measuring the duration of visual perceptions. In this instance, as in a former paper, I am impelled to emphasize the difference between visual perceptions and sensations. This is especially necessary in studying the subject of visual persistence. Vision is always accompanied by a perception, and it is the duration of the perception that ordinarily interests us. The duration of a visual sensation apart from a perception cannot be measured, for it has no existence. It seems preferable, therefore, to speak of the duration of visual perceptions

rather than that of sensations. An added reason for this distinction is that I have shown, in the paper which I published in 1901, that the duration of visual perceptions may be decidedly affected by psychological conditions, but comparatively little by the intensity of the luminous stimulus. This would appear to indicate that the persistence of vision is largely due to the activity of the higher visual centers. It is evident that, if we could count the number of the bands in the rod experiment, knowing the rate of revolution of the disc, we could estimate their duration. This cannot be done satisfactorily, but it is quite possible to measure the duration of the perceptions by employing another method. When one end of the rod or strip which casts the shadow is held so that the end coincides with the prolongation of the axis of the disc, and the other end is made to describe a circle slowly, the bands are radial. If the rate of revolution of the rod is properly adjusted the bands can be made to cover the entire disc. The slowest rate of revolution of the rod which enables us to complete the circle of bands is the measure of the duration of the perceptions, and I have found this to be about four-tenths of a second. The essential part of the mechanism employed for this purpose is illustrated by Fig. 10.



The small black circle in the center of Fig. 10 represents a section of a cylinder to which one end of a slender rod is attached, which is seen projecting radially over the middle of the sector. The axis of the cylinder is the prolongation of that of the disc. The cylinder with the rod may be revolved independently of the disc about the common axis, and in either direction. The rate of revolution of the rod may be varied continuously within the required limits. If the rod revolves slowly, clockwise, while

the disc revolves with sufficient rapidity, the subjective bands will be radial, as in Fig. 11. If the disc revolves clockwise, the first radial line in this figure represents the rod, and the remaining four radii the dark bands which we are studying. When the rod revolves more rapidly, the bands may be made to extend completely around the circle, as is shown in the last figure. In Fig. 11 the band farthest from the rod is the one formed first, and is about to disappear. As the rate of revolution of the rod increases the oldest band recedes from the rod until it is diametrically opposite. When the bands cover more than a half circle, and the rod revolves still faster, the disappearing band approaches the rod on the other side of the disc until it finally coincides with it, as in Fig. 12; the rate of revolution of the rod is then the measure of the duration of the band. When the rate of revolution of the rod remains constant, the number of bands seen will depend on the rate of revolution of the disc. If the disc revolves at constant speed the radial bands will become more widely separated when the rate of revolution of the rod increases, but it will also be noted that a greater number of bands will be seen when they extend entirely around the disc. This depends upon the fact that the mind can note and retain a larger number of the visual impressions which give rise to the bands when they are widely separated in space. As I have noted before, the duration of visual perceptions is chiefly affected by psychological factors.

The successful use of the apparatus just described requires some practice and concentration of attention, and, in using it, it is especially important to resist the tendency to follow the revolutions of the rod with the eye. As the perception becomes less vivid before it entirely disappears, an allowance has to be made for this fact in determining when the shadows are distributed entirely around the disc. It requires but little experience with the apparatus to demonstrate that with 150 revolutions of the rod per minute, and probably with a rate somewhat less, the perception of the shadow persists during an entire revolution. This indicates a duration of at least four-tenths of a second. This result is in accordance with those obtained with other

methods, and recorded by me in a former paper. It has been amply proven that this duration is much longer than has been generally supposed, and that it depends comparatively little on the intensity of the stimulus. The factors that abridge the duration are especially those that interfere with attention. By the use in a dark room of the apparatus to which I have given the name of Prism Stroboscope, which was described in the paper which I published in 1901, it has been shown that luminous impressions may last three-fourths of a second. In a dark room all distracting influences are at a minimum, and there is the greatest possible contrast between the image and the field. In the shadow experiments just described the whole field of vision is receiving simultaneously many impressions which cannot be ignored, and which lessen contrast.

The attentive study of the results of the band experiment sheds light on a number of important visual processes. Visual inhibition is most clearly demonstrated in the bands. In the case of a white sector on a black ground the fusion color is a dark gray, but the radial bands are nearly or completely black. By reducing the angular extent of the sector it is possible to lessen the duration of the retinal shadow to a thousandth of a second, or even to a period much less than that. When this is done the resulting gray is very dark, but the bands are distinct and much darker. Unless the illumination is very intense there will be complete fusion of the black and white without flicker, nevertheless, the withdrawal from the light of the sector of the shaded portion of the retina for a thousandth of a second is sufficient to inhibit the previous white constituent of the gray, which persists in the unshaded region with nearly undiminished intensity. In other words the previous gray sensation is inhibited, and a perception of a black band corresponding to the retinal shadow remains. This inhibition of a sensation can be shown even more strikingly by using a colored sector of considerable brightness on a colored ground of feeble luminosity. The brighter color will then be inhibited, and the bands will have the color of the ground, or one closely approaching it.

Another condition, known in physiology as the refractory state, plays a part in the production of the bands. It should be remembered that when the retinal shadow lasts only the thousandth of a second, the perception of the shadow continues for four-tenths of a second, or possibly a little longer. I have even succeeded in obtaining with a very narrow white sector distinct bands due to shadows that lasted less than three-thousandths of a second, and the duration of the bands was as given above. It is, therefore, easy to prove that under appropriate conditions the perception may outlast the retinal shadow more than a thousand times. During the period of four-tenths of a second the part of the retina upon which the shadow fell was stimulated at least twelve times by the repeated passages of the sector without evoking any sensation in the corresponding part of the visual field. The perception of the shadow made that part of the visual field refractory to stimulation by the reflected light of the sector. This indicates that in the complicated processes of vision sensations which interfere with the clearness of perception may be suppressed or prevented. Visual perceptions are due to the activity of the higher visual centers, and it is in these centers that it is most probable that inhibition and the refractory state occur. The evidence of the band experiments is in accord with that furnished by my early experiments, and authorizes the same conclusions.

If we replace the revolving rod with a black disc having a narrow radial slit, and view the other disc as it revolves rapidly, through the slit, gray or colored bands will be seen if the object disc is well illuminated. These bands correspond to the dark ones due to the revolving rod, and their duration may be estimated in the way that has been described. This duration seems to be slightly longer than with the rod, or about one-half of a second. A white or colored sector may be used on a black or colored ground. The distinctness of the bands depends on the size of the slit and of the sector, the difference in the luminosity of the sector and ground, and the illumination of the object-disc. In order to insure sufficient illumination, the object-disc, revolving at the rate of about 1,800 revolutions a minute, should be placed

20 or 30 cm behind the slit, and near a window, or a lamp in a darkened room if artificial illumination is desired. It is desirable that the slit should be adjustable, and it should be viewed at a distance of one or several meters. This makes a convenient arrangement for measuring the persistence of vision, and for proving that this persistence is very constant under widely varying conditions.

FROM THE AUTHOR'S LABORATORY
MONSON, MASSACHUSETTS.

A COMPARISON OF THE FECHNER AND MUNSELL SCALES OF LUMINOUS SENSATION VALUE

By Elliot Q. Adams

A critical study of the Munsell scale of (luminous sensation) value has been made by Priest, Gibson, and McNicholas.[1] Their results "verify in a remarkable manner the consistency of the Munsell values for different hues . . . considering the uncertainties of heterochromatic photometry which were necessarily involved in Munsell's work." They establish that "the squares of the Munsell value numbers are directly proportional to the reflection of sunlight"—as might be expected from the construction of the Munsell photometer, which employs an Aubert diaphragm,[2]—and object that "the implication that values, read directly as the diagonal of the shutter, are proportional to sensation in the sense of Fechner's law is quite wrong." Since it is well known[3] that Fechner's law is only an approximation to the law relating brightness and sensation, and an equation which constitutes a closer approximation has recently been published,[4] it will be well to examine, in the light of this newly-found relation, both the Fechner and the Munsell (or Stefanini)[5] scales.

Before proceeding to a quantitative comparison, it will be well to point out anew[6] the difference between the range of valid-

[1] I. G. Priest, K. S. Gibson, and H. J. McNicholas, Technological Paper No. 167, Bur. Stds. (Sept. 1920) "An Examination of the Munsell Color System. I. Spectral and Total Reflection and the Munsell Scale of Value."

[2] H. Aubert, Grundzüge der physiologischen Optik, pp. 489, 547. Leipzig, W. Englemann, 1876.

[3] And is explicitly conceded by the authors on page 29 of the reference in footnote 1.

[4] E. Q. Adams and P. W. Cobb, J. Exp. Psych., *5*, pp. 39-45; 1922.

[5] A. Stefanini, Nuov. Cim. (3) *22*, p. 97; 1887 (for sound). Atti della R. Acc. Lucc. di Sc. Lett. ed Arti., *25*, pp. 383-400 (for light and weight). The Stefanini formula is the special case of Plateau's formula $E = kR^{\epsilon}$, in which $\epsilon = 0.5$.

[6] See for example, A. Elsas, Wundt's Philos. Stud. *4*, 162-79 (1888), also E. B. Titchener, Experimental Psychology, Instructor's Manual, Quantitative, pp. 210-32. §29. New York, Macmillan, 1905. The method of equal sense distances: historical and critical.

ity of Weber's law, and the range over which Fechner's sensation law holds. This comparison can be made more concrete by the analogy of the measurement of current by a tangent galvanometer[7] provided with an assortment of shunts. Such an instrument gives greatest percentage precision when the scale reading is in the neighborhood of 45°, and the percentage precision at that (or any other constant) angular deflection is the same whenever the shunt is selected so that the current measured gives that deflection. Hence with a sufficiently varied supply of shunts the percentage precision may be made constant over the range covered by the shunts, that is:

$$\Delta I = \frac{\Delta s}{b} I = cI \qquad (1)$$

where ΔI is the least detectable increase in current, I; b and c constants, and Δs the least perceptible change in scale reading, s. At the same time *with any one shunt the* scale reading, s, is related to the current by the equation:

$$I = I_m \tan s \qquad (2)$$

where I_m is the current which for that shunt gives a scale reading of 45° and hence maximum precision. Yet the equation obtained by integrating (1), in the form $Ids = bdI$;

$$s = b \ln I \qquad (3)$$

does not hold at all, for over the range of validity of (1), the scale reading is always near to 45°. Now if I represent light intensity and s sensation, (1) and (3) become Weber's and Fechner's Laws respectively.

The analogy is, of course, not perfect, for while equation (1) in the form

$$\Delta B = cB \qquad (4)$$

holds over a considerable range of brightness, the equation[8]

[7] This instrument is chosen because its deflections follow a simple mathematical law and remain finite as the current is indefinitely increased. For the sake of continuous variation of the shunt resistance a slide wire might be used.

[8] Equation (2) of the article referred to in footnote (4), based on the assumption that visual impressions are transmitted along each fiber of the optic nerve by a series of impulses whose effect depends only on their frequency,—the All-or-None hypothesis of Keith Lucas, "The Conduction of the Nervous Impulse," p. 9, London; 1917. Cf. also L. T. Troland, J. Opt. Soc. Am., *4*, p. 160; 1920.

which has been found to relate sensation to brightness *at constant adaptation*, (the analog of galvanometer readings with a given shunt) is not of the form of equation (2), but is

$$s=\frac{B}{B+k} \tag{5}$$

where k is a constant dependent on the state of adaptation, being equal to the brightness at which photometric precision is a maximum (for the given state of adaptation).

s expresses sensation as a fraction of the maximum possible range of sensation; if it is desired to express it in other units, a coefficient, a, must be inserted in equation (5). Similarly if sensation is to be measured from any other point of reference than the sensation corresponding to (physically) complete blackness, a term, s_o, for that sensation, must be introduced into equation (5) which thus becomes

$$s=s_o+a\,\frac{B}{B+k} \tag{6}$$

The relation between sensation and brightness thus assumes an infinite number of forms according to the value of k. Since in equation (6) s becomes independent of B at both limits, $k \doteq 0$ and $k \doteq \infty$, the law of variation in these limiting cases may be found as follows. For $k \doteq 0$, i.e., for dark adaptation

$$s=s_o+a\,\frac{B}{B+k}=s_o+a\,\frac{(\ 1\ )}{1+k/B}\doteq s_o+a\left(1-\frac{k}{B}\right)=(s_o+a)-\frac{ak}{B} \tag{7}$$

while for adaptation to infinite brightness, $k \doteq \infty$,

$$s=s_o+a\,\frac{B}{B+k}\doteq s_o+\left(\frac{a}{k}\right)B \tag{8}$$

that is, the sensation approaches in the first case a linear function of the reciprocal of brightness, in the second a linear function of brightness itself.

Many of the other formulas which have been found empirically are special cases of the Plateau equation

$$s=k'B^{\epsilon} \tag{9}$$

where ϵ is an exponent lying between 0 and 1, and k' a constant.

If ϵ be given the value $\frac{1}{2}$, equation (9) becomes the Stefanini[8] equation, on which the Munsell scale is based:

$$s = k'\sqrt{B} \tag{10}$$

If sensation be not measured from the sensation produced by the physical absence of light, a term, s_o, must be added in equation (9) as in the case of equations (5) and (6), giving

$$s = s_o + k'B^{\epsilon} \tag{11}$$

If ϵ be made equal to unity, this becomes the Merkel[9] proportionality law:

$$s = s_o + k'B \tag{12}$$

which is identical with (8).

When ϵ approaches O, (11) becomes

$$s = s_o + k'e^{\epsilon \ln B} \doteq s_o + k'\,(1 + \epsilon \ln B) = (s_o + k') + k'\epsilon \ln B \tag{13}$$

that is, Fechner's law, which is, therefore, the other limit of the Plateau equation.

It will be noted that equations (5) and (9),—and hence also the equations derived from them,—retain their form if B be measured in other units, provided the appropriate changes are made in the constants of the equations. Since for any constant illumination the relation between sensation and test object *albedo*[10] (the brightness relative to that of a perfect diffusely reflecting surface similarly illuminated) will depend upon the albedo of the surroundings but will be independent of the illumination,—within the range of brightnesses for which Weber's law holds,—equations (4) to (13) may be made the same for the relation between sensation, s, and test object *albedo*, as for the relation between sensation and brightness. In what follows the symbol, B, and the term "albedo" will both signify *test object albedo*.

Priest, Gibson, and McNicholas give in their Fig. 16, (on p. 31), a comparison of the Merkel, Munsell, and Fechner scales, made to coincide for Nos. 1 and 9 of the Munsell scale.

[9] Julius Merkel, Wundt's Philos. Stud., *4*, pp. 117-60, 251-91, 541-94, 1888; *5*, pp. 245-91, 499-557, 1889; *10*, pp. 140-59, 203-48, 369-92, 507-22 especially p. 517; 1894.

[10] This term is used habitually by astronomers in stating the reflecting powers of the planets.

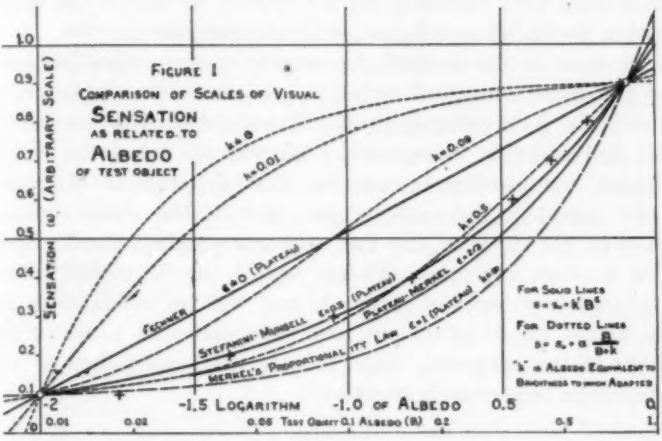
TABLE 1. *Relation Between Sensation and Test Object Albedo, According to Various Scales and Theories*

Sensation Arbitrary Scale	Scale No.	Albedo according to: Munsell Scale: meas. by Priest *et al*	Munsell Scale: Theoretical (Stefanini)	Merkel $\epsilon=1$	Fechner $\epsilon=0$	Plateau $\epsilon=2.3$	Adams and Cobb $k=0$	Adams and Cobb $k=0.01$	Adams and Cobb $k=0.09$	Adams and Cobb $k=0.50$
0.0	0	....	.00	.....	.0058	....	.0089	.0078	0	.....
0.1	1	.018	.01	=.01	=.01	=.01	=.01	=.01	=.01	=.01
0.2	2	.041	.04	.11	.0173	.0578	.0114	.0128	.0225	.0522
0.3	3	.090	.09	.21	.0300	.126	.0133	.0165	.0386	.102
0.4	4	.161	.16	.31	.0520	.211	.0159	.0215	.0600	.162
0.5	5	.234	.25	.41	.0900	.310	.0198	.0290	=.09	.234
0.6	6	.343	.36	.51	.156	.420	.0261	.0412	.135	.325
0.7	7	.465	.49	.61	.270	.542	.0386	.0656	.210	.441
0.8	8	.602	.64	.71	.468	.670	.0736	.1267	.360	.595
0.9	9	.772	.81	=.81	=.81	=.81	=.81	=.81	=.81	=.81
1.0	10	....	1.00	.91	1.403	.958	.....	.....		1.130
Equation No. ...			10	8,12	13	11	7	6	6	6
	s_0 ...		0	.09	.938[b]	.0549	.91[b]	−.72	0	.0738
Coefficient[a]	.....		1	1	2.147[a]	1.972	−.0081	1.64	1	1.336

[a] The coefficient, in the appropriate equation, of B, log B or 1/B respectively. For Fechner's Scale the coefficient is that for *common* logarithms.

[b] These values are that for s_1, the sensation for unit *albedo*, in the case of the Fechner scale, and that for S_∞, the sensation for infinite relative brightness, for the limiting case $k=O$, since in both s_0 is infinite.

A similar comparison is given in Table 1 and Fig. 1 of this paper. The figure differs from that of Priest *et al*,—besides giving curves for several other formulas than the three named,—in two respects; sensation (or value) has been plotted against log albedo, instead of log albedo against sensation, and the theoretical curves have been made to agree at numbers 1 and 9 of the *theoretical* Munsell scale, (equation 10) while the *albedo* of the Munsell papers, as measured by Priest *et al*, is indicated by crosses. Solid lines indicate formulas derived from that of Plateau (equations 9 to 13) for the indicated values of the exponent ϵ, dotted lines show the relations given by the equation of Adams and Cobb



(equations 4 to 8) for various values of k, the brightness to which the eye is adapted, (expressed in *albedo* units). The Merkel proportionality law, being a limiting case of both the foregoing, is indicated by a dashed line.

From the figure it can be seen that all the other curves lie within those representing the limiting cases of constant adaptation to zero and infinite brightness, respectively; hence, by an appropriate constancy or variation in the state of adaptation during the measurements, any of the relations shown could be obtained experimentally. Again, it will be noted that with adaptation to the

geometric mean ($k=0.09$) of scale numbers 1 and 9 (of the theoretical Munsell scale) the relation between sensation and albedo approximates the scale of Fechner, whereas with adaptation to the arithmetic mean[11] of the same brightnesses ($k=0.41$), it agrees well with the Stefanini equation on which the Munsell scale is based. It is noteworthy that the actual Munsell scale agrees fully as well with the equation of Adams and Cobb (for $k=0.5$), as with the Stefanini equation.

In view of the criticisms by Priest, Gibson, and McNicholas it may be well to point out the *physical basis* of the Munsell scale. Its numbers represent the *amplitude* of the light waves from a diffusely reflecting surface relative to that of the waves from a *perfect* diffuse reflector, similarly illuminated.

In view of the marked dependence of the subjective scale of (luminous sensation) value on the state of adaptation, it is doubtful if the axiom of Priest, Gibson and McNicholas (p. 29): "It will probably be agreed by all who are interested in the subject and consider it carefully, that the steps in the value scale should be apparently equal; that is, the visual contrast between the cards of any two adjacent numbers should equal that between any other adjacent two,"—can be applied to the grading of the series of grays. It may well be preferable to use the actual albedo of the surfaces, since this scale is one of the *limits* of the subjective scale, namely that approached as the adaptation brightness is increased.

SUMMARY

1. Only if the state of adaptation of the eye is maintained constant, is it proper to speak of luminous sensation as a function of brightness.

2. With constant adaptation, k, the functional relation between sensation, s, and brightness, B, is well represented by the equation of Adams and Cobb.

$$s=\frac{B}{B+k} \tag{5}$$

[11] The curve for $k=0.41$ in the equation of Adams and Cobb has not been represented on Fig. 1, but it can easily be seen that it would lie only slightly above that for $k=0.50$.

3. All the equations connecting sensation and brightness are of such a form that,—within the range of validity of Weber's law,—the relation between sensation and test object *albedo* may be made independent of the absolute level of brightness (for any constant illumination of test object and surroundings) but will depend on the albedo of the surroundings to which the eye is adapted.

4. Between numbers 1 and 9 of the Munsell scale of (luminous sensation) value, the sensations of an eye adapted to a brightness corresponding to the arithmetic mean[11] of the albedo of those scale numbers (i.e., 0.41) approximate the values of the Munsell scale.

5. Within the same limits, the sensations of an eye adapted to a brightness corresponding to an albedo of 0.09,—the *geometric* mean of the albedos corresponding to Munsell scale numbers 1 and 9,—approximate the values of the Fechner scale.

6. In view of the marked dependence of subjective value on the state of adaptation of the eye, grays should be rated according to their *albedo*, which is a physically determinate property.

NELA RESEARCH LABORATORIES
CLEVELAND, OHIO
AUGUST, 1922.

INSTRUMENT SECTION

THE MEASUREMENT AND SPECIFICATION OF OPTICAL CHARACTERISTICS IN PROJECTOR PERFORMANCE

By G. W. Moffitt

In order to facilitate further improvement in the art of optical projection it has become desirable to definitely recognize those optical characteristics of projector performance which should be the basis for judging the merits of any projector of whatever construction. The measurement and specification of these characteristics should also be considered and a system adopted which will not only make possible a critical study and comparison of the various types of projector but will also make the results of such study and comparison generally available instead of leaving this information entirely with a few.

Optically the performance of projectors may vary in definition, in screen illumination, and in quality. In addition to these there are certain other phenomena—such as shutter effects and vibration—that affect the optical performance but which are due to causes entirely mechanical. For the presence of these defects the optical system should not be blamed, but nevertheless account must be taken of them in the specification of the optical characteristics of projector performance. Moreover, it is obvious that a consideration of the fine points in the optical performance of projectors may be profitably taken up only after the objectionable mechanico-optical effects have been reduced to an inoffensive degree.

Definition

The first demand on the optical system of a projector is that it produce a well-defined screen image. Usually the projection lens is held entirely accountable for the definition, but unless it is properly supported by the machine good results may not be obtained even with the best of lenses. If the film is not maintained in a proper fixed position relative to the optical system, or if it is not allowed to remain stationary during the projection

intervals, then the performance will be unsatisfactory no matter how excellent the lens may be. Moreover, there are certain optical relations between condensing system and projection lens that may affect the definition to some extent. This matter will be considered in a subsequent paragraph.

Of recent years the increasing use of the motion picture in the home and in the school at rather short viewing distances and with small screens has resulted in the production of apparatus using miniature films. On the basis of conservative study of this matter the conclusion is justifiable that there is a proper place for films of this kind. In some cases there has been a tendency to crowd on the magnification and make these miniature outfits project a picture much larger than was necessary for satisfactory results. This practice requires a film magnification as high—sometimes higher—than that required for standard projection of very large pictures where the viewing distance is great. It means that the miniature film demands a degree of correction in the projection lens hitherto not called for in the standard sizes. If these small films come into general use a very superior type of projection lens will be required.

Formerly the specifications for good projection lenses did not tax the resources of the designers. But now the increasing demands for higher efficiency have made refinement necessary. Relative apertures have been increased until—as seems to be the opinion of many—further advance in this direction is limited by the decreasing depth of focus in its relation to the mechanical limits in maintaining the film position and contour. More and more we find designers turning their attention to improvement in definition, in transmission, and in quality of image. Along with these advances in projection lens design the need has arisen for a workable system of measurement and specification of projection lens performance and of the optical characteristics of projector performance in general.

No doubt there are those who would advocate the use of precision lens bench tests as the basis for the specification of definition in projector performance. But while it is true that the lens bench is a very useful aid in the critical study of the

performance of lenses intended for use as visual or as photographic objectives it does not necessarily follow that it is universally applicable, except in the hands of an experienced worker. Because of the power of the instrument to show up the defects of a lens system there is sometimes a tendency to judge too critically when examining a lens in this manner, and the necessity for making allowances in the interpretation of the results gives an opportunity for bias and temperament to affect the judgment. Of course this ability to reveal image defects makes the instrument indispensable to the progressive designer who wants to know *in what way* the lens is failing. Lens bench tests are made with a uniform distribution of light over the aperture of the objective whereas in actual projection this might be, but usually is not, the case. In fact the intensity distribution over the area of a projection lens aperture is usually very far from uniform. This lack of uniformity is such that the performance of a projection lens will often be better than a lens bench test would lead one to expect. Obviously then, the projection lens should be tested in position and under actual working conditions. Illuminating systems differ considerably in the matter of uniformity of intensity at the lens. Therefore a lens possessing considerable zonal aberration may show up better on some projectors than on others. As higher and higher optical efficiency is attained the opening of the projection lens will become more and more uniformly and completely filled. Here again, it may be noted, the conditions for improved efficiency demand better correction of the aberrations of the projection lens.

The definition tests should be made when the machine is completely adjusted for showing a picture of normal size at normal screen distance. If the sum total of optical performance as affected by the mechanical defects is desired the machine should be running at full speed while readings are being made. If it is suspected that the mechanical effects are of considerable magnitude and that therefore the test may not be a fair one for the optical system the readings should also be made with the mechanism at rest. Thus whether the tests be made with the machine in motion or stationary depends on whether one is

testing *the individual complete machine* or *the optical system of the machine*.

A suitable standardized plate, such as is shown in Fig. 1, of sufficient rigidity to prevent warping may be placed in the film-gate and the focus set for the best delineation of the central hole. It is then a simple matter to find the position, with respect to the screen plane, of the primary, the secondary, and the best image, and also the distortion for any or all of the field points. This may easily be done with a white card and a meter scale.

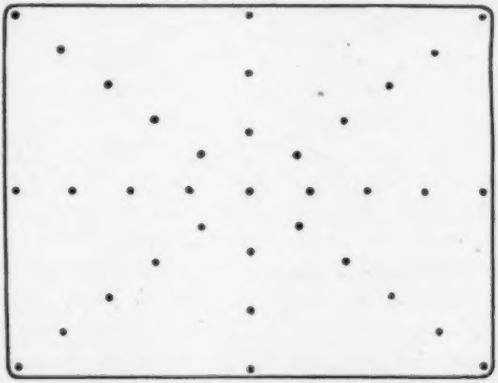


FIG. 1. *Showing the arrangement of pin-holes in a proposed film-gate stencil plate suitable for the study and measurement of the aberrations of a projection lens.*

If any of the desired images fall behind the screen one may place the focus for the central spot at a suitable small distance in front of the screen and all of the images which it is desired to measure may thereby be made accessible. Thus all that is necessary to map the field is the standardized plate, a meter stick, and a white card to serve as a search screen. Coma, as usual, refuses to submit to numerical specification and we must be content with a more or less qualitative statement regarding it.

In case the quality of the axial definition is in question one may resort to a consideration of the best image size—after making proper allowance for the geometrical magnification—or to resolving power tests carried out in the usual way although

resolving power tests of this kind made under conditions of uniform flux density at the lens should be accepted here with some reserve for the reasons already pointed out. The design of a satisfactory resolving power test slide to fit into the film-gate is a matter that might well receive considerable attention.

SCREEN ILLUMINATION

The measurement of screen illumination is comparatively easy but deductions from such measurements should be drawn cautiously for the contrast rendering ability is sometimes affected by gain in illumination. Illumination gained at the expense of contrast rendering ability may be no net gain at all. In order that the illumination measurements may be complete it is necessary to determine the intensity, the uniformity, the screen size and distance. Measurements should always be taken with the machine running for there are certain types of shutter that have a vignetting action. Since the screen brightness always depends on the characteristics of the screen itself it is obvious that *screen illumination* and not *screen brightness* should always be measured when the performance of the projector alone is under consideration. Machines with freak shutters having translucent blades, or blades otherwise diffusive, offer some difficulty in the determination of effective screen illumination and unless care be exercised the conclusion may be reached that the effective illumination is higher than is actually the case, the apparent gain being only diffuse light that can do nothing but fog the screen and degrade the quality of the projected picture. On the other hand perforated flicker blades, or flicker blades regularly transmitting a portion of the light, may actually increase the effective illumination to some extent.

Uniformity of screen illumination is, perhaps, best expressed by stating the ratio of the illumination at the various screen points in question to that at the center of the screen. It would be advisable to select and standardize a certain set of screen points to be used in this connection. A stencil to fit the film-gate could be made that would indicate on the screen the points at which to make illumination readings. Fig. 2 shows a proposed stencil

of this kind that is easily made of fine wires stretched on a frame. The illumination is to be read in the small squares, these areas being so chosen that a good idea of the distribution is obtained. Whatever the arrangement of such a set of selected points may be they should be so chosen that any lack of symmetry either along horizontal or vertical lines will be apparent. There are a number of factors entering into the question of the best distribution of screen illumination which would seem to indicate that a uniform distribution, or even a symmetrically vignetted distribu-

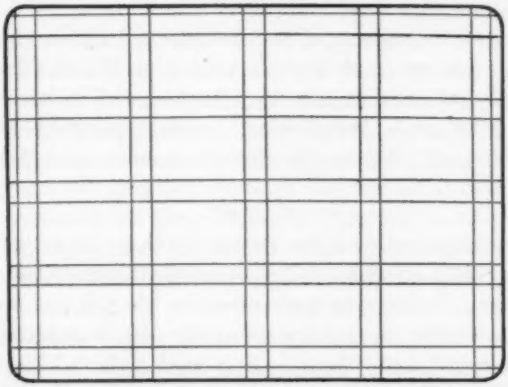


FIG. 2. *Stencil of fine wires for use in the measurement of screen illumination. Readings are made on the centers of the small squares.*

tion, is not generally the best, and that there is no one type of distribution that is best for all installations. Therefore a rather complete statement of the distribution is desirable, in order that one may estimate correctly the performance of the projector in any given auditorium.

The results of a screen illumination test should be accompanied by a statement of screen size and distance, for both the distribution of the light and the intensity may be changed in a manner other than the size of the screen would lead one to expect. The change in focus for different distances may necessitate slight readjustments of the illuminating system and so change slightly the effective diaphragming. And, of course, the performance of a given machine for a given picture size with a projection

lens of one focal length should not be taken to apply to the performance of the same machine with a projection lens of the same make but of another focal length the distance being changed so that the picture size is maintained. It is well known that the illumination falls off for a given picture size as the focal length of the projection lens is increased. But it does not seem to be so well known that there is no fundamental theoretical reason for this and that present-day machines have this characteristic simply because manufacturers have gotten into the habit of making them that way.

Qualitative statements as to color effects, local patchiness, and striations not revealed by the values of illumination at the standardized points should also be included in the complete statement of screen illumination, although no projector can be considered good if it is not possible to eliminate such defects.

QUALITY

Quality in projection is determined by those characteristics of a projector which affect its ability to render faithfully the contrasts of the film. It deserves more attention than it has received in the past. Under this head we find the defects generally known as "flare-spot" and "flare." Scattered light which does not come through the lens but reaches the screen by other paths need not be considered here for such light can always be controlled.

Fortunately nearly all good projection lenses are of a type especially free from inherent flare-spot and flare, that is, the design is such that reflection images are few in number and advantageously located. But with the newer trend toward higher relative aperture and higher film magnification a tendency may be noted in some quarters to depart from the time-tried types and exploit others neither so well suited to give high transmission of image-forming light nor so fortunate in the small number and desirable position of their reflection images.

Flare-spot.—This defect is noticeable whenever an image of a strongly radiating point is formed by reflection between some of the lens surfaces, the image so formed falling near the screen. The result is that irregular areas of fog and more or less concen-

trated spots of light may play about on the screen, their position, intensity, and motion depending on the character of the film being shown. They may be more conspicuous with some condensing systems than with others.

To detect the presence of flare-spot in a projector place the definition test slide in the film-gate precisely as in the definition test. Any other opaque slide having a number of small distributed holes will serve. If splotches of light appear among the screen images of the holes the system is afflicted with flare-spot. Obviously nothing more than a qualitative statement can be made as to the extent to which this defect is present.

Flare.—Flare is usually due to out-of-focus flare-spot. That is, the light which in some cases goes to make a flare spot is so much out of focus that it is quite uniformly distributed over the screen. Another source of degraded contrast in projection is found in poorly polished surfaces and surfaces that are not clean. Obviously all these causes of flare are more marked in their effects the greater the number of air-glass surfaces.

If the machine is equipped with a shutter having any of its blades translucent, or otherwise diffusing, the tests for flare should be made with the machine running. Solid blades with edges blackened would not be expected to have any noticeable influence on the quality of the picture, but shutters with some of the blades of diffusing material add to the total illumination only by the amount of scattered light they throw onto the screen with the result that the picture is degraded thereby.

The specification of flare may best be made by stating the maximum contrast which the projector is capable of rendering in the most unfavorable case, that is, when a small opaque object is depicted on a fully illuminated screen. The necessary measurements can be made by placing a small opaque and well-blackened disk or strip in the film-gate as shown in Fig. 3, and then determining the ratio of full illumination on the screen adjacent to the image of the disk or strip to that in the image itself. Care must be taken to see that the measuring instrument is capable of measuring the contrast obtained with the best lenses. Furthermore it must be used in such a way that there is no question of

diffusely reflected light getting into the dark half of the photometer field. A statement of the full illumination and of the picture size should also be included.

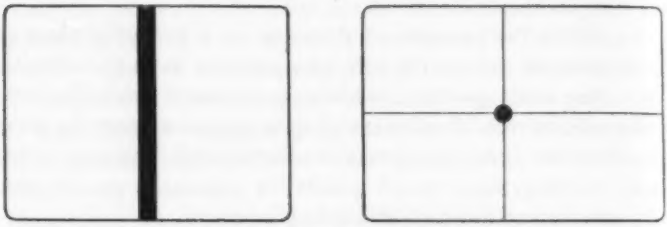


FIG. 3. *Proposed stencils for flare determination.*

CONCLUSION

By the use of some system of measurement and specification, such as that here outlined, the optics of projection may be placed on a definite basis quite as readily as has already been done for the photographic objective. The improvement of the performance by the improvement of one or more of the component parts is, of course, a matter for the designer and its consideration is beyond the scope of this paper.

FRANKFORD ARSENAL,
PHILADELPHIA, PA.
JULY, 1922.

AN ELECTRON TUBE TUNING FORK DRIVE*

By E. A. Eckhardt, J. C. Karcher and M. Keiser

In view of the increasing use of tuning forks, both in research and in industry, the maintenance of tuning fork vibrations has become a matter of some practical importance. Applications of the tuning fork (1) as a sound source; (2) as a small scale time standard; (3) as a frequency or pitch standard; (4) as a current interrupter, and (5) as a synchronizer or speed-controlling device, are suggestive of the general wide field of usefulness of the tuning fork as a research and engineering instrument.

The invention of the electrically-maintained tuning fork is probably due to Lissajous,[1] although frequently ascribed to Helmholtz.[2] The method used by both involves the periodic opening and closing of the electrical circuit by the vibrating forks. Many modifications of detail involving this general principle, however, are to be found described in the literature. The difficulties encountered in the microphonic behavior of the interrupting contact were made the basis of the microphonic method of maintenance by Appleyard.[3] Until 1919 the interrupted contact and the microphonic methods for maintaining tuning fork vibrations electrically were practically the only ones available, the former being almost universally used.

During 1919 papers appeared by Abraham and Bloch[4] and Eccles and Jordan[5] describing methods for maintaining tuning fork vibrations electrically without circuit interruptions, by means of electron tube circuits. Contemporary work by the present authors[6] led to similar results. The purpose of this paper is to describe apparatus and circuit arrangements which we have found advantageous and to point out advantages of this method.

It is well known that the maintenance of tuning fork vibrations by means of circuit interrupters is more difficult the higher the

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frequency of the fork. Beyond 300 cycles the difficulties are very great.[7] Helmholtz[8] vibrated a series of eight tuning forks, whose respective frequencies were 120 and integral multiples thereof, by means of an interrupter fork of frequency 120 cycles per second. This procedure of driving a high frequency fork by means of a sub-harmonic interrupter fork has recently been used by Curtis and Duncan[9] for maintaining the vibrations of a 500-cycle fork in a tuning fork chronograph. The driving and driven fork frequencies must be adjusted to have a precise multiple ratio. This adjustment is quite critical and is not easily maintained, particularly if the frequency ratio is high. We have applied the electron tube drive to a large number of forks in the frequency range from 50 to 2,000 cycles. There has been no occasion to attempt higher frequencies, but the experience with the 2,000-cycle fork indicates that it is entirely feasible to go higher in the frequency scale.

The Principles of the Electron Tube Circuit Tuning Fork Drive

The electron tube circuit in general consists of a plate or power circuit and a grid or control circuit. Both are completed through the electron tube and have common circuit elements therein. These circuits may be coupled externally in such a manner that the total action becomes regenerative. This is the case when the coupling is such that the current rise in a plate circuit is followed by a potential rise of the grid which, operating through the tube, will serve to augment the current rise in the plate circuit which started the action. The current thus generated is oscillatory because of the nature of the circuit and soon reaches a maximum which is determined by the closeness of coupling and the characteristics of the vacuum tube.[10]

A characteristic form of regenerative circuit is shown in Fig. 1. The external coupling between the grid and plate circuits is here inductive. If the circuit constants are suitably related[11] such a circuit will act as a generator of electrical oscillations the frequency of which may be controlled by an adjustment of the circuit constants.

Forced vibrations of a telephone diaphragm or a tuning fork are obtained if the windings of the telephone receiver or the driving electromagnet of a tuning fork are included in the plate circuit of such an oscillation generator or coupled to it through a transformer. By adjustment of the oscillator frequency to resonance relatively large vibration amplitudes may be obtained. The

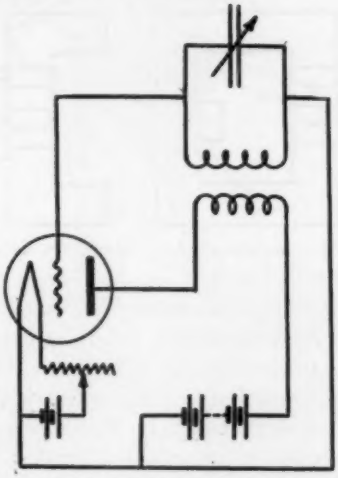

FIG. 1. *Regenerative electron tube circuit.*

maintenance of tuning fork vibrations in this manner is not satisfactory, however, because it is more or less difficult to maintain the frequency of an oscillator circuit sufficiently constant while yielding sufficient power to assure an adequate amplitude of vibration.

As pointed out by Eccles and Jordan the coupling in a regenerative circuit may be wholly provided by a vibrating mechanical system, the interaction between the regenerating circuit elements being due to the mechanical motion, or the coupling may be partly due to the mechanical system, and partly electrical. In the apparatus developed by the authors the mechanical coupling predominates and the inductive coupling between the grid and plate circuits is adjustable. The arrangement is shown

schematically in Fig. 2. Moving the laminated iron yoke which carries the grid coil up to or away from the driving electromagnet changes the electrical coupling within suitable limits. The variability of the coupling thus provided is of importance in adjusting the circuit for any one fork or in making the same circuit available for driving forks of different frequencies. The complete

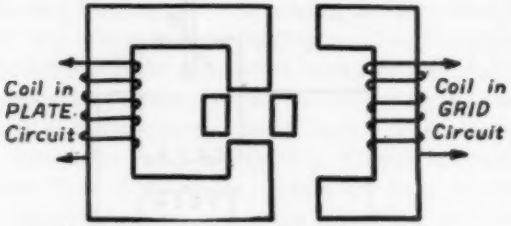

FIG. 2. *Schematic diagram of tuning fork drive.*

circuit in idealized form is shown in Fig. 3, and Fig. 4 shows an equipment with a 500-cycle tuning fork mounted for driving.

The functioning of the circuit is in brief as follows: The closing of the plate circuit (with filament bright) starts the plate current through the driving electromagnet. The fork-prongs are pulled together and thereupon vibrate feebly. Inspection of Fig. 2 will

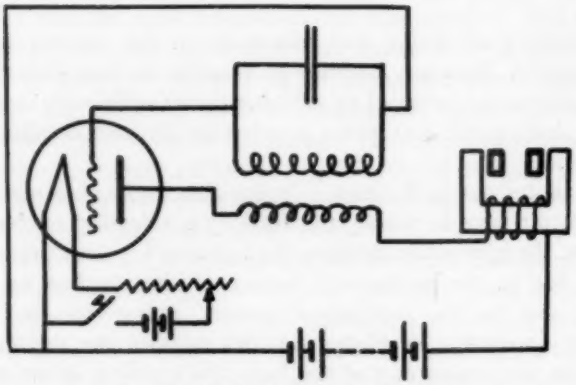

FIG. 3. *Idealized tuning fork drive circuit.*

indicate that this feeble motion of the fork has a relatively large effect on the reluctance of the magnetic circuit. The reluctance

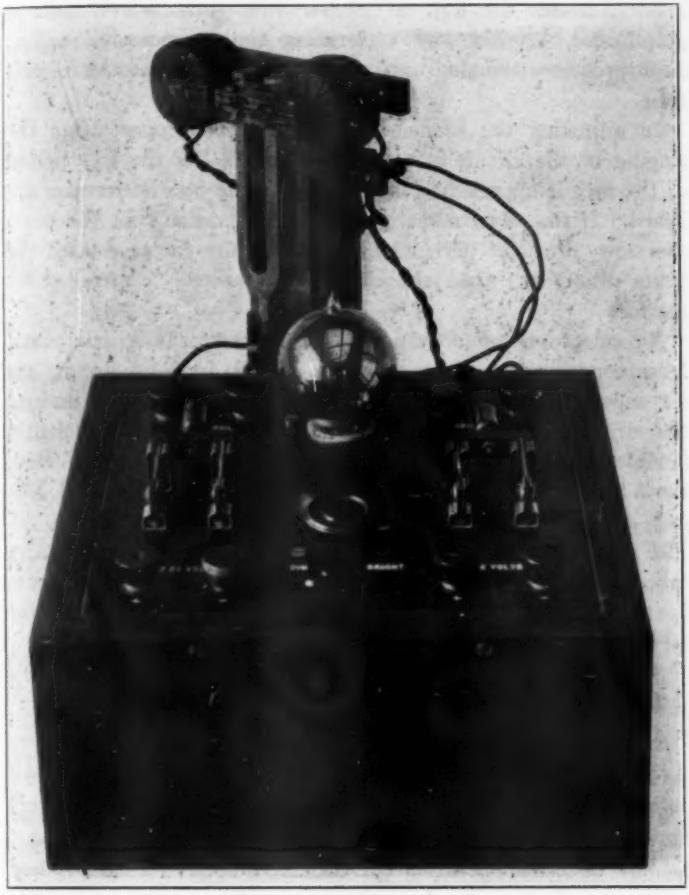

FIG. 4. *500 cycle tuning fork unit.*

changes are determined by and are in synchronism with the free period of the fork. Corresponding variations in the plate

current result, which by virtue of the coupling existing between the plate and grid circuits are regeneratively amplified. The alternating component of the plate current, as its growth progresses, causes the fork to vibrate with progressively greater amplitudes, in this way acelerating the regenerative action. This process continues until limited by the properties of the tube.

In adjusting the filament current to the proper value the singing of the circuit is a valuable guide. As the free period of the fork is approached beats between the two frequencies are heard. If the adjustment is sufficiently close so that the beats are slow, the beat period will automatically increase until the beats disappear, when the circuit is completely controlled by the fork.

A condenser is connected across the grid coil for tuning purposes. Mica condensers of the type generally used in radio circuits are far superior to paper condensers for this purpose. In general the driving arrangement illustrated in Fig. 4 is sufficiently flexible so that commercially available condensers may be used. For a 500-cycle fork the plate coil has 1,200 turns of No. 28 B. & S. gage silk-enamel wire and the grid coil 3,000 turns of No. 30. The tuning condenser has a capacity of 0.25 microfarad. Any tube of 5-watt rating, providing a steady plate current of approximately 50 milliamperes with 220 volts on the plate is suitable for use in the circuit.

Fig. 5 shows a circuit arrangement in which all the necessary power for operation is derived from a 220-volt power circuit. The diagram is practically self-explanatory. The 3,300-ohm rheostat permits the adjustment of the grid to positive mean potentials, which is sometimes advantageous. The ammeters shown in the filament- and plate-circuits are very convenient because once the operating currents have been established they facilitate adjustment when the power line voltages are not uniform.

The drive discussed for tuning forks is adaptable for maintaining the vibrations of a diaphragm at its natural frequency.

Fig. 6 shows the method of drive applied to a Webster phone. Fig. 7 shows the phone with the cylindrical resonator case removed. A small piece of iron is mounted on the diaphragm. Its position is adjusted to correspond to that of the right hand prong of the tuning fork in Fig. 2. The position of the driving magnet M is adjustable by sliding along the guide rods R_1 and R_2.

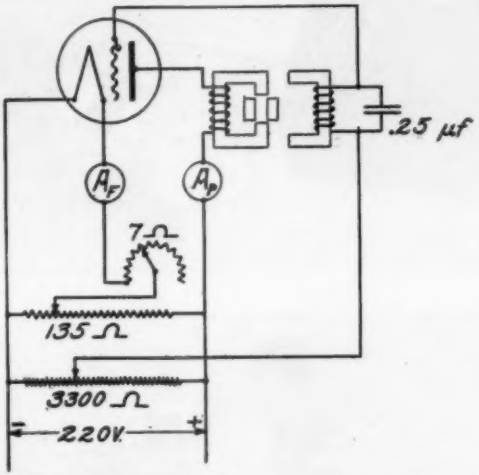

FIG. 5. *Circuit for operation from 220 volt power supply.*

The clamp screws C_1 and C_2 hold it firmly in the chosen position. The grid magnet G is mounted behind the driving magnet to slide on the same guide rods, the relative position being adjustable during operation by means of the adjusting screw S.

This type of tuning fork drive is especially available for use with tuning fork chronographs where precision of frequency is a matter of great importance.[12]

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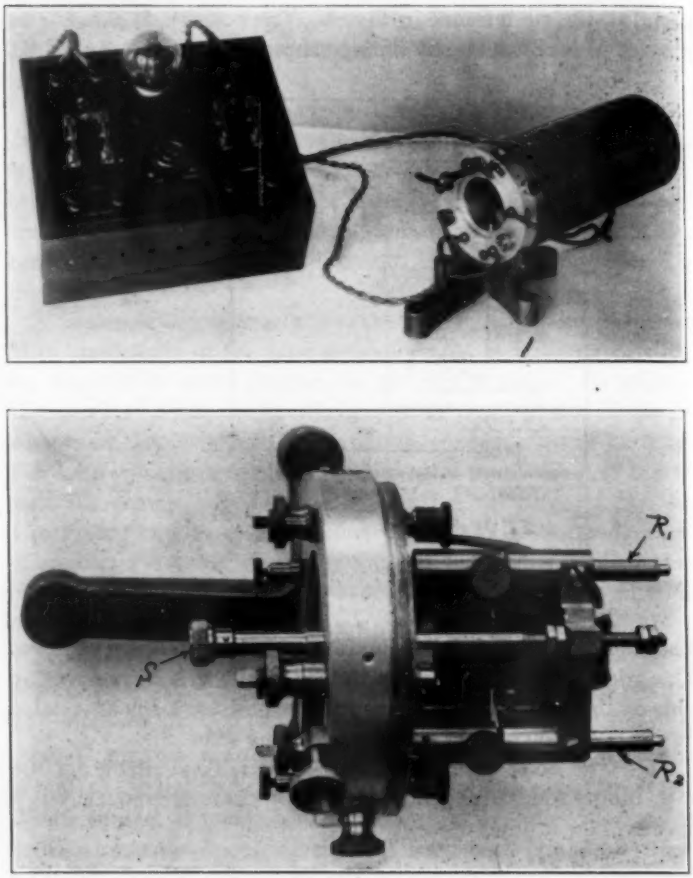
3. R. Appleyard, On a method of driving tuning forks electrically. Proc. Phys., Soc., *10*, pp. 288-290. Phil. Mag., (5) *28*, p. 490; 1889.

4. Abraham and Bloch, C. R. pp. 1197, 1198, June 16, 1919; Jour. d. Phys., Ser. V, *9*, pp. 225-233; 1919.



FIGS. 6-7. *Webster Phone with electron tube circuit drive of diaphragm.*

5. Eccles and Jordan, The Electrician, *82*, p. 704, June 20, 1919.

6. Eckhardt, Karcher, Keiser, Phys. Rev., Ser. 2, *17*, pp. 535-536; 1921.

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A LABORATORY HYPSOMETER*

By E. F. Mueller and T. S. Sligh, Jr.

A very simple form of hypsometer, consisting of a nearly closed space into which steam from a boiler can be admitted, will serve to maintain, in the steam space, a temperature which differs, at most, by a few hundredths of a degree from that corresponding to saturated steam at atmospheric pressure. The well known Rudberg or Regnault hypsometer is a very simple piece of apparatus and is capable of serving the requirements of all but the most precise thermometric measurements.

Of the various more elaborate forms of hypsometers which have been devised for precise measurements two were designed at the International Bureau, while most of the remainder originated at the Reichsanstalt. In the Chappuis hypsometer[1] which was an improved and simplified form of a very elaborate instrument devised by Pernet,[2] the design was determined almost entirely by the requirement for facility in changing the position of the thermometer from vertical to horizontal, while in the steam. In this hypsometer, the steam is lead from the boiler through piping to a tube in which the thermometer is placed, thence into an annular space surrounding the tube, and thence to the condenser. A water manometer is used to measure the difference between the pressure in the steam space and that of the atmosphere. A copy of this instrument has been in use for many years at the Bureau of Standards, and the only feature of it to which objection might properly be made, is the water manometer.

Thiesen, Scheel and Sell[3] describe a very elaborate hypsometer, which was a modification of an earlier instrument described by Pernet, Jaeger and Gumlich.[4] The hypsometer as modified con-

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[1] Described by Guillaume, Trav. et mem. du Bur. Int., *5*, p. 39; 1886.

[2] Trav. et Mem. du Bur. Int., *1*, p. B-15; 1881.

[3] Wiss. Abh. der Phys. Tech. Reichsanstalt *2*, p. 138; 1895.

[4] Wiss. Abh. der Phys. Tech. Reichsanstalt *1*, p. 87; 1894.

sisted of a large gas heated fire tube steam boiler, from which the steam was piped through a pressure regulator, to the space in which the temperature was to be measured. In the bottom of this space was a water seal, through which the entering steam was made to pass in order to ensure saturation. The steam escaped from the steam space through a second pressure regulator. An important feature was the manometer for measuring the excess pressure in the steam space. The one limb of this manometer which was in contact with the steam, was surrounded by water which was kept boiling violently by means of steam supplied by a separate boiler. The other limb of the manometer included a water surface of large area.

In the description of this apparatus, the errors due to possible superheating, and to incorrect measurement of the pressure in the steam space, are emphasized, and it is evident that elaborate measures were considered necessary to eliminate such errors. It is reported that the apparatus functioned satisfactorily.

A later apparatus described by Guitzmacher[5] has the general appearance of the Regnault hypsometer, but differs from it in provision of special means to eliminate superheating. The steam from the boiler flows through a number of short tubes, through a water seal forming the bottom of the thermometer space, and thence past the thermometers. A number of water manometers were connected for indicating the excess pressure of the steam, but evidently it was not considered necessary to boil them as in the apparatus previously described.

It does not appear necessary to refer to all of the various other forms which may be found described in the literature. It will be sufficient to refer to the hypsometer used by Henning & Heuse[6] in their recent determination of the expansion of gases. Steam was generated in a small boiler, heated by an electric heating coil immersed in the water. The steam was piped to the top of the annular space surrounding the thermometer space, flowed downward in the annular space and up past the thermometer

[5] Wiss. Abh. der Phys. Tech. Reichsanstalt, *3*, p. 259; 1900.
[6] Zs. für Physik *5*, p. 295; 1921.

and thence to the condenser. The excess pressure was read on a water manometer. All parts were thoroughly insulated to avoid fluctuations of temperature due to drafts.

The last two forms of apparatus are noteworthy as indicating a tendency to depart from the somewhat monumental form attained in earlier instruments. It is also noteworthy that the last apparatus while omitting the very elaborate precautions observed in the design of earlier forms, was used with platinum resistance thermometers, which would have made it possible to detect errors and irregularities so small as to escape detection entirely in work with mercurial thermometers.

The above brief review will indicate that in the design of hypsometers, in addition to the essential precaution of steam jacketing for the space in which the thermometer is placed, which is the feature that makes the distinction between the Rudberg or Regnault hypsometer and an ordinary tin can, the refinements which have been emphasized are (1) avoidance of superheated steam in the space around the thermometer, (2) accurate measurement of the pressure in this space, (3) provision for securing constancy of pressure and temperature in this space. To these the authors would add purity of material as an essential feature, to be attained by rapid and thorough removal of air from the steam.

In designing a new hypsometer[7] for general laboratory use, it appeared that an improvement on the instruments already described could be obtained by introducing the steam into the thermometer space and the surrounding annular space at the top, allowing the steam to flow downward in parallel in the two spaces. This arrangement secures steam jacketing of the thermometer space, avoidance of superheat in the steam, since the boiler is located at a distance and ample cooling surface can be provided between boiler and thermometer space, and eliminates entirely the necessity for a water manometer to measure the excess pressure of the steam. The density of steam being less than that of air, a column of steam flowing downward in a pipe

[7] Briefly described in Jour. Wash. Acad. *11*, p. 167; 1921.

open to the air at the bottom, is stable, while if the steam is flowing upward in a pipe, stability can only be attained by restricting the escape of steam sufficiently to cause the pressure in the pipe to exceed that of the atmosphere. The downward flow of the steam also greatly facilitates the removal of air from these spaces.

The details of construction will be evident upon reference to the photograph, Fig. 1 and the schematic line drawing, Fig. 2. Electric heating was chosen for convenience. The boiler is made of a brass tube 5 cm in diameter and 20 cm high, the outside reservoir serving to maintain a nearly constant water level. The steam pipe from the boiler is a 13 mm brass tube, which enters the steam space around the thermometer tangentially. The small boiler makes it possible to heat up rapidly, and its small cross section and the small steam pipe provide for relatively rapid steam flow in these portions thus facilitating rapid removal of air. The upper 5 cm of the thermometer space are uninsulated to provide surface for condensation, while the remainder of the space is provided with an air jacket which is apparently sufficient to prevent fluctuations due to drafts. The steam after passing through the thermometer space, escapes into the air or may be condensed and returned to the boiler.

All parts in contact with steam or water were tinned. The heating coil consists of two sections which may be connected by means of the switch, either in series or parallel. With the coils in series on a 110-volt circuit the input is about 125 watts which is sufficient to maintain just a trace of steam escaping into the air. Increasing the input up to 640 watts produced no determinable change in the indications of a resistance thermometer in the steam, thus indicating the absence, both of superheating and of excess pressure.

A series of steam point determinations with a resistance thermometer, using both the new hypsometer and the Chappuis hypsometer, indicated no systematic difference between the two, although the precision was slightly in favor of the new instrument. Fortuitous errors of about 0.005° persisted and are apparently due

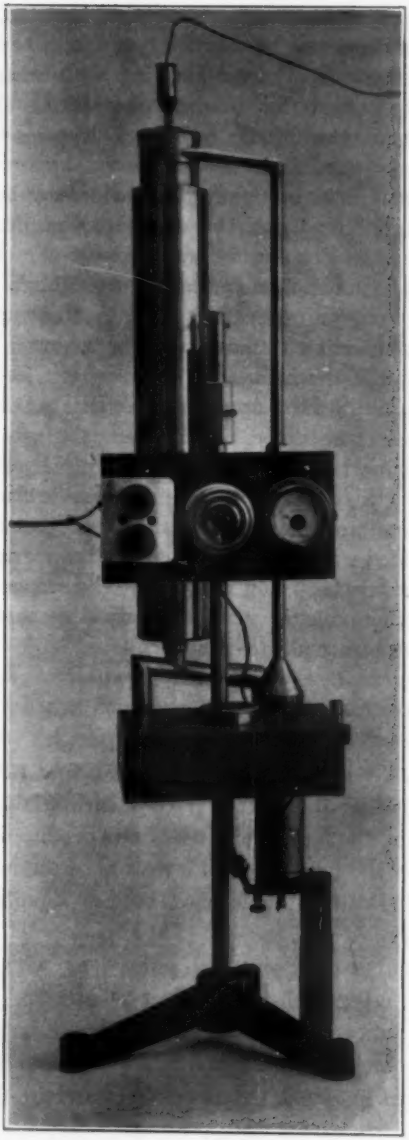

FIG. 1. *Laboratory Hypsometer*

to irregular fluctuations in atmospheric pressure or to errors in measurement of barometric pressure indicating that improve-

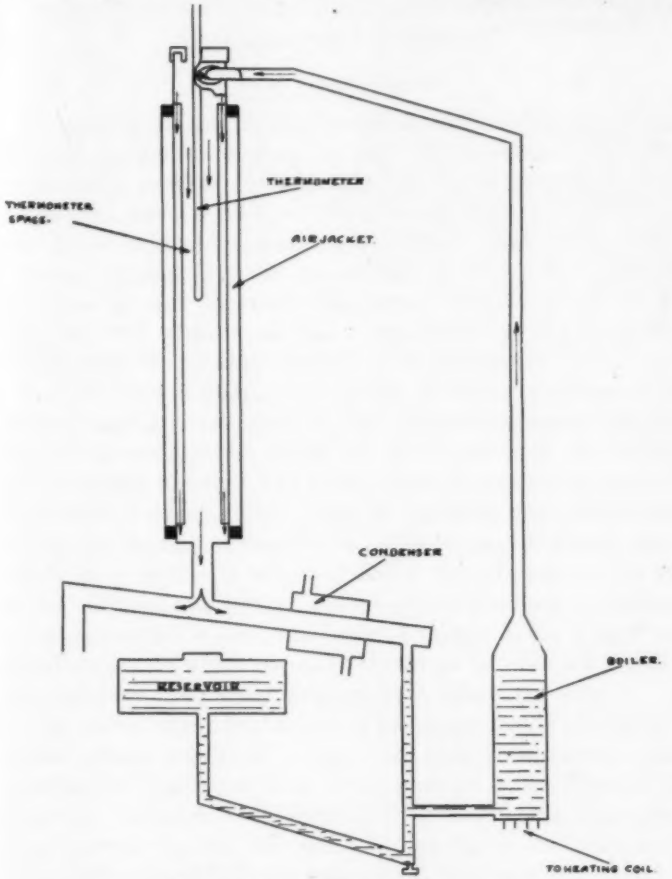

FIG. 2. *Schematic Diagram of Hypsometer*

ments in this respect will require the use of a closed system and better temperature control of the barometer.

While there is no indication that results hitherto obtained with other hypsometers are in error, it is believed that the principles

applied in the design of the present instrument can be used to advantage in the construction of new equipment.

BUREAU OF STANDARDS,
WASHINGTON, D. C.

A HIGH TEMPERATURE REGULATOR FOR USE WITH ALTERNATING CURRENT

By Howard S. Roberts

For many processes, in the factory as well as in the laboratory, elevated temperatures must be held within narrow limits for considerable periods. Where this can be accomplished automatically a part, at least, of the personal element is eliminated and more satisfactory results are secured. Sometimes, where electrical heating is used, the voltage of the power available fluctuates to such an extent that hand regulation is out of the question, and without automatic regulation no processes that involve close temperature control can be employed.

In a recent publication[1] the author described a furnace temperature regulator for direct current resistance furnaces, depending on the temperature coefficient of resistance of the heating element of the furnace. The heating element formed one arm of a Wheatstone bridge and its change of resistance with temperature caused the heating current to be increased and decreased alternately by a system of relays replacing the galvanometer of the bridge. Adams and White[2] had previously described a regulator operating on this Wheatstone bridge principle; while Haagn[3] has described one in which the ratio of current to voltage-drop thru the furnace is maintained constant by a differential relay.

The present paper has to do with the adaptation of the author's direct current regulator for use with alternating current, thus meeting the limitation that direct current is not everywhere available. A possible advantage of using alternating, rather than direct current is that the effect of leakage on the readings of thermoelements at high temperatures is considerably reduced.

[1] J. Wash. Acad. Sci. *11*, pp. 401-409; 1921.

[2] Phys. Rev., *14*, pp. 44-48; 1919.

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OPERATION OF THE REGULATOR

Since the fundamental principles underlying the operation of regulators of this type have been discussed elsewhere,[4] the present paper will include only those matters peculiar to the alternating current apparatus.

The various parts are essentially the same as were used in the direct current apparatus, except that a more satisfactory relay has been developed and a simple rectifier placed in the galvanometer circuit, thus making possible the use of the original direct current galvanometer.

The functions of the various parts of the regulator will be sufficiently clear from the diagram, Fig. 1, to admit of postponing their detailed description until the operation of the regulator has been described.

We shall ignore, for the moment, the effect of self induction in the heating element and in the other resistors that together form the bridge, and assume that at a given instant the temperature of the heating element, and therefore its resistance, is such that the bridge is balanced. There is now no current flowing from *a* to *b* thru the galvanometer circuit. If the temperature of the heating element rises, its resistance increases and an alternating emf is set up between *a* and *b* which may be considered as in phase with the current flowing thru the bridge; if the temperature falls from this higher value, the emf *ab* passes thru zero and becomes 180° different in phase; i.e., it reverses just as would be the case if the bridge were supplied with direct current.

The current for the galvanometer passes thru a rectifier which, as will be shown later, causes a direct current to flow thru the galvanometer in one direction when the emf *ab* is in phase with, and in the opposite direction when the emf *ab* is 180° from the bridge current. Thus, the current thru the galvanometer passes thru zero and reverses as the temperature of the heating element passes a particular value, determined by the positions at which the contact arms *a* and *b* have been placed. The boom of the galvanometer (in the figure) tends to swing toward the right with rising temperature and toward the left with falling temperature.

[4] Roberts, op. cit., pp. 405-407.

The actual control of the temperature is brought about by cutting in and out a fixed series resistance, the "regulating resistance" in Fig. 1, as the temperature falls and rises, respec-

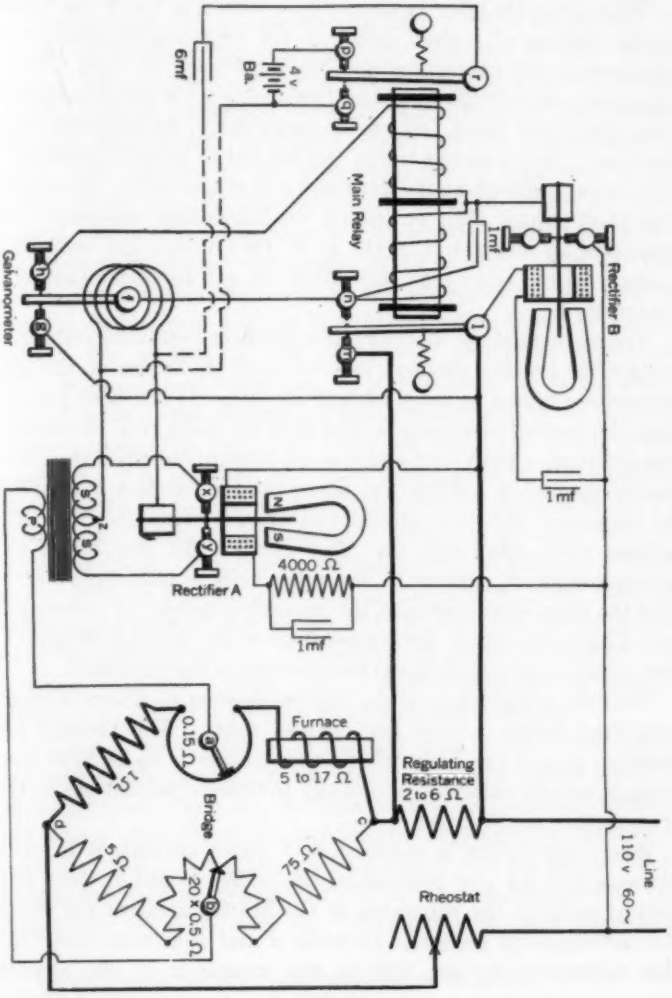

FIG. 1

ively. The outside rheostat must therefore be set to such at value that the temperature will fall when this resistance is in, and rise when it is out.

With the main relay in the position shown in Fig. 1 the "regulating resistance" is short circuited, the temperature is rising and the boom of the galvanometer is moving toward the right. When the boom reaches the right hand contact *g* a circuit is completed thru the right hand coil of the main relay, causing the relay to close. This does two things (we are not at present considering the contacts shown at the left hand end of the relay): it removes the short circuit thru *m* around the regulating resistance; and provides an alterantive path, at *n*, thru which the energizing current of the relay may flow after the galvanometer contact *g* has opened.

The removal of the short circuit around the regulating resistance causes the heating element to cool, and the boom of the galvanometer begins to move toward the left. As has been pointed out, the current continues to flow thru the main relay because of the alternative path thru *n* and is not interrupted when the boom leaves contact *g*. When the boom reaches contact *h* a circuit is completed thru the second coil of the relay; the two coils oppose each other and the relay opens. By this action the whole supply of current to the relay coils is interrupted at *n*, and the short circuit around the regulating resistance restored at *m*. The latter causes the temperature of the heating element to rise, completing one cycle of the operation of the regulator.

Thus the temperature of the heating element oscillates between two fixed limits, a very few degrees apart. The thermal lag between it and the body whose temperature is to be held constant is usually sufficient practically to prevent oscillations in the temperature of the latter.

Where the bridge is supplied with direct current, it is quite obvious that for any position of the sliders *a* and *b*, there is a critical value of the resistance of the heating element for which the difference of potential between *a* and *b* is zero, and that this difference changes sign as the resistance of the heating element varies from a value slightly less to one slightly greater

than the critical value. In the case of alternating current, this potential difference can only be zero in the particular cases where the reactances as well as the resistances of the four arms of the bridge are in proportion, or where the resistances are in proportion and the reactances are zero. As is customary in dealing with alternating currents, the potential *ab* may therefore be represented by a vector whose magnitude and direction will, in general, both vary with the temperature of the heating element. If, however, we resolve this vector into a component in phase with, and one at 90° from the potential impressed on the bridge (from the potential *cd*), there will, as before, be some critical value of the resistance of the heating element for which the former (in-phase) component of *ab* is zero. Further, this component must change sign as the resistance of the heating element varies from a lower to a higher value than the critical value. On separating this component selectively from the 90° component and rectifying it, we obtain a pulsating direct current whose sign changes as the temperature of the heating element passes the particular value for which the sliders *a* and *b* are set. The selection and rectification may be accomplished by means of the synchronous vibrator described herewith or, for example, by means of a commutator driven in synchronism with the line voltage. The same result might also be obtained without rectification by making use of the electro-dynamometer principle (alternating current galvanometer).[5] In any case the apparatus must be adjusted so that the component of *ab* selected by it is in phase with the voltage impressed on the bridge, in order to eliminate the effect of reactance in the bridge resistors.

So long as the reactances of the four arms of the bridge remain unchanged, the magnitude and direction of the vector potential *ab* is definitely fixed by the resistance of the heating element; but if they vary, because of a change in frequency, for instance, *ab* must also vary. It has already been shown that the galvanometer is affected only by changes in the power component of *ab* (the component in phase with *cd*). The effect of small changes in

[5] As suggested in the author's previous paper, op. cit., p. 407.

reactance on the power component is small compared to the effect on *ab* as a whole; and where, as will usually be the case in practice, the ratio of reactance to resistance is itself low, the absolute change in *ab* as a whole is so small that the effect, of changes in reactance, on its power component can be neglected.[6] We should therefore expect the temperature of the furnace or bath to be practically unaffected by any accidental variation in the frequency of the alternating current supply.

This expectation was verified, in the case of a resistance furnace, by comparing the temperature at which the regulator held it when supplied with alternating current, and with direct current. The temperature in the first case was 1081.1° and in the second, 1081.6°, a difference of 0.5°. This corresponds to the maximum possible frequency change of 100 per cent.

DESCRIPTION OF APPARATUS

The details of the bridge, indeed of the apparatus as a whole, will depend on the particular conditions under which it is to be used, but it may be worth while to take up certain general considerations, as well as to describe one form of the regulator.

The heating element should be made of some material having a rather high temperature coefficient of resistance, and should not oxidize readily at the temperature at which it is used. Copper and iron are satisfactory at low temperatures; nickel up to about 500°. The alloy "alumel" is said to have a high temperature coefficient and might prove satisfactory up to 800° or 900°. At still higher temperatures platinum is probably best.

Although the various resistance coils need not be wound non-inductively, the use of iron in their construction had better be avoided; this is particularly true of the bridge coils. The fine adjustment *a*, Fig. 1, may be a piece of heavy resistance wire bent into an arc of a circle, and the contact a short piece of the same wire set radially on a pivoted arm. In the case of the coarse adjustment *b*, a dial switch whose points are connected to the resistances is to be preferred to a rheostat whose slider makes contact directly with the turns of wire on a resistance coil: in

[6] All of this can be shown rather simply by means of a vector diagram.

the latter case the contact arm is very likely to shunt one or more turns of the wire and, as the contact resistance changes, the setting of the bridge is disturbed.

As the apparatus is capable of responding to changes of 1 part in 20,000 in the resistance of the heater, the apparatus should be so constructed that the change in bridge setting from all other causes is considerably less than this. This means that the bridge resistances must be made from one or another of the low temperature coefficient alloys such as manganin, therlo, constantan, ideal, advance, etc.; the temperature coefficients of nichrome and chromel are a little too high. The connections within the bridge circuit should be of one of these alloys or of heavy copper and the joints should be soldered wherever possible. In order to prevent electrical or magnetic leakage from affecting the galvanometer, the connections to it should be thoroly insulated and the transformer must not be located too close to any of the resistance coils.

In the apparatus used by the author the values of the various resistances and capacities are those given in Fig. 1. These are chosen for use with any suitable heating device whose resistance, when in use, lies between 5 and 17 ohms. Power is supplied at 110 volts, 60 cycles, and the main relay has a capacity of about 10 amperes.

The galvanometer is the same instrument used in the direct current temperature regulator described elsewhere.[7] It is a Weston "Model 30, five binding post galvanometer," having a resistance of 50 ohms and a period of about 2/3 second. It is essentially a millivoltmeter in which the pointer is replaced by an arm carrying a contact button of "iridium alloy" which makes contact with one or the other of two contact screws, depending on the direction of the current in the coil. A potential of 1 millivolt across the coil terminals causes the contact button to move about ½ mm.

When the apparatus is running continuously, each of the galvanometer contacts may be called upon to operate as often as

[7] Roberts, op. cit., p. 404.

40,000 times per day, and, unless the current flowing thru them is very small they quickly become pitted and stick. With a current of about 10 milliamperes, however, the contacts seldom stick for more than a few seconds before breaking loose, and will usually run for several days without attention. Contacts of Acheson graphite have been found very satisfactory; they need to be cleaned much oftener than the alloy contacts but have the advantage that they may be cleaned off with a nail file without interrupting the regulator.

Sticking of the galvanometer contacts when it does occur causes temperature fluctuations, so that for very precise control of the temperature some means must be adopted to prevent it. The inductive device as used in the direct current apparatus[8] can not be used here, therefore we make use of the energy stored up in a condenser by a small dry battery.

This device is shown at the left hand end of Fig. 1, and for simplicity that portion of the relay is represented in the diagram as an additional armature r. With this armature in the position shown, there is a circuit from the battery B thru the 6 mf condenser and the galvanometer back to the battery, so that the condenser is charged to the potential of the battery. When the armature of the relay is attracted this connection thru the battery is broken and another made around the battery, at q, causing the condenser to discharge thru the galvanometer. When the armature drops back against p, the condenser is again connected to the battery and the charging current flows thru the galvanometer but in the opposite direction to the discharging current. The polarity of the battery is such that there is in either case a momentary deflection of the galvanometer away from the particular contact thru which it has just caused the main relay to operate.

In practice the contacts seldom fail to close the circuit promptly. When they have failed, it has invariably seemed to be due to a coating of charred dust and the trouble has disappeared when this was removed.

[8] Roberts, op. cit., p. 402.

The rectifier, Fig. 2, consists of a reed *R* of clock spring, clamped to a support at one end. The reed passes thru a coil *C* of about 1500 turns of No. 36 B. & S. gage (0.13 mm) copper wire wound on a hollow bobbin of vulcanized fiber. The free end of the reed swings between the poles of a permanent magnet *NS*. The coil *C* receives current from the line and induces an alternating magnetic field in the reed. This causes the end of the reed to be attracted alternately by the north and by the south pole of the magnet *NS*, so that the reed vibrates with a pitch equal to the line frequency. A short distance from the clamped end the reed carries a contact button of molybdenum which may make contact with either

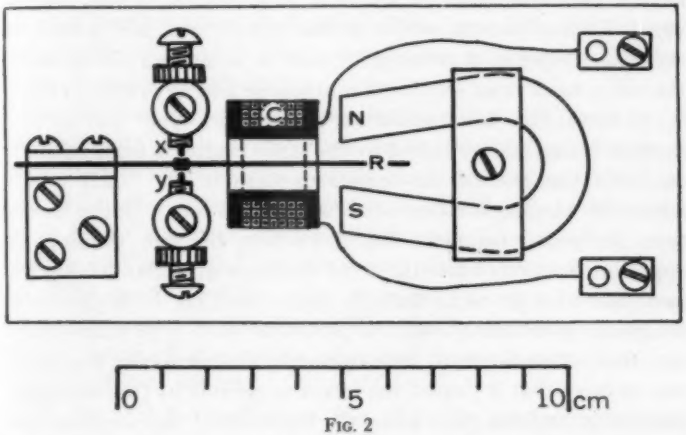

FIG. 2

of the molybdenum tipped contact screws *x* and *y*. Thus a circuit is closed thru one contact screw during the positive half of the reed's cycle and thru the other contact screw during the negative half.

The emf to be rectified is that of the split secondary of the transformer shown in Fig. 1. Here the emf's measured from the center *z* to the two ends are opposite in phase. Thus if the difference in phase between the vibration of the reed and the emf of the secondary as a whole is either 0° or 180°, a unidirectional emf is set up between the reed and the middle tap *z* of the

split secondary and a pulsating direct current flows thru the galvanometer. The direction of this current depends on whether the difference in phase referred to above is 0° or 180°.

If the difference in phase between the vibration of the reed and this emf is not 0° or 180°, it can be shown that, while the *instantaneous* value of the rectified emf may change sign during its cycle, the *average* value is equal to that which would be obtained by rectifying only that component of the secondary in phase with or 180° from the vibration of the reed.

The transformer has a primary winding having 1,000 turns of No. 24 B. & S. gage (0.51 mm) copper wire and two secondaries having 1400 turns each of No. 28 B. & S. gage (0.32 mm) copper wire. Each of these is wound in two equal parts, one on each of the longer sides of a rectangular core of laminated silicon steel; the latter has a cross section 12 mm square and an opening 25 mm by 45 mm. This is a transformer of the "core type" and for this purpose is much less affected by the stray magnetic fields emanating from the rest of the apparatus than is the "shell type," where the winding is all on one branch of the core. In the former type the useful magnetic flux flows thru the two windings in opposite directions while the stray fields set up a flux having the same direction in each; thus the emf's resulting from the latter neutralize each other. With a particular shell type transformer the effect of the magnetic field caused by the regulating resistance was so large that it caused the relays to operate as this resistance was cut in and out even when the transformer was disconnected from the bridge.

Owing to the fact that the resistance of the primary circuit of the transformer is not low in comparison with its reactance, the phase of the secondary emf leads that of the emf impressed on the primary. It has already been shown that it is desirable that the vibration of the reed be in phase with the particular secondary emf that would be induced by a primary emf in phase with that impressed on the bridge. This may be brought about by inserting a condenser and parallel resistance in series with the magnetizing coil *C* and the phase may be adjusted by varying the amount of parallel resistance. The correctness of this adjust-

ment may be tested while the regulator is running by placing a bar of iron in or close to any one of the bridge resistances. This changes the inductance of that particular coil and, unless the adjustment of the reed is correct, causes a halt in the otherwise steady clicking of the relay.

In the case of Rectifier *B* it seemed better to dispense with a transformer (chiefly because no suitable transformer was available) and to replace it with a condenser in parallel with the relay. Under these conditions the condenser supplies current to the relay during that half of the cycle when the rectifier contact is open. This particular rectifier works satisfactorily with only a condenser in series with the magnetizing coil, and under these conditions the relay receives about 80 per cent of the current it would take if connected directly to a direct current line of the same voltage as the alternating current line.

The "main relay," which is operated by direct current obtained from Rectifier *B*, is a converted "main line" telegraph relay. There are two spools of about 20 000 turns each of No. 40 B. & S. gage (0.08 mm) enameled copper wire on either leg. The two spools nearer the yoke, connected in series, form the left hand winding in Fig. 1; and the other two, also connected in series, the right hand winding. The resistance of each of the two windings is about 11 000 ohms. The separate windings must be thoroly insulated from each other and from the iron core on which they are wound.

The left hand armature of Fig. 1 is the original armature found on the relay, while what is represented in the figure as the right hand armature is in reality a strip of spring brass, attached to the original armature and insulated from it. This spring should be rather stiff and the contact screws should be so adjusted that there is considerable pressure on contact *m* when the relay is open. These contacts, *l*, *m*, and *n*, are of molybdenum, 1.5 mm square and may be used to break currents up to 10 amperes at 40 volts. For heavier duty it is better to break the current by means of a second, heavier relay operated by this one.

Where the occasional sticking of the galvanometer is not an objection, the contacts *p*, *q*, and *r* may be omitted along with their

battery and condenser shown in Fig. 1. In this case the original armature may be used for *l*.

If the current thru the heating element exceeds 10 amperes, or the temperature must be controlled for periods longer than about 24 hours without supervision, it will usually be more satisfactory to turn over the control of the heating current to a second, more rugged relay. This may be one of the stock relays ("remote control switches") built for heavy duty, or a telegraph relay may be fitted with two or more sets of contacts connected in parallel through suitable resistances. By the latter means the shunt around the regulating resistance may be removed in several steps and the emf, available to cause sparking at the various contacts, reduced to a negligible amount. In either case the energizing current for the second relay passes through the contacts *l* and *m* of the differential relay in Fig. 1.

The use of this second relay reduces the current through the contacts of the main relay to ½ ampere or less, so that the main relay may be adjusted to operate on a much smaller current. This slows down the deterioration of the galvanometer contacts, and the apparatus may be run for longer periods without attention.

As was the case with the direct current apparatus a variable shunting resistance may be connected in parallel with one of the arms of the bridge in order to vary the temperature of the furnace or bath continuously in either direction.[9] In the present apparatus this takes the form of a rheostat giving steps of 10 ohms from 1000 to 2000 ohms and connected in parallel with the 75 ohm coil of the bridge. The heating and cooling curves obtained with this are not quite linear, but are entirely satisfactory for most purposes.

PERFORMANCE

The sensitivity of the regulator is about the same as that of the author's direct current apparatus described elsewhere. The hot resistance of the winding of a furnace wound with platinum and insulated with magnesia is held constant to the equiva-

[9] Op. cit., p. 409.

lent of $\pm 0.1°$ at 1000-1400°. The actual temperature of the furnace falls less than 1° per day when it is maintained about 1200°. At 1400° it may fall as much as 10° per day. This fall in temperature is quite steady and seems to be due to changes in either the heating element or in the insulation of the furnace itself.

Although the regulator has been used with an oil bath, no numerical data of its performance with such a bath are available. There seems no doubt, however, that with a heater of large area and with effective stirring, temperature fluctuations should not exceed 0.05° at 200°.

GEOPHYSICAL LABORATORY,
CARNEGIE INSTITUTION OF WASHINGTON,
WASHINGTON, D. C., JUNE 12, 1922.

AN INTEGRAPH BASED ON PARALLEL DOUBLE TONGS

By Vladimir Karapetoff

An integraph[1] is a mechanical device which draws a differential or integral curve to a given curve. Referring to Fig. 1, let $y=f(x)$ by a given curve plotted against OO as the axis of abscissae, and let $z=\phi(x)$ be another curve plotted against NN as the axis of abscissae. Let e and e' be two points on these curves corresponding to the same x. In other words, let the origin on NN be shifted by the amount A to the right, with respect to the origin on OO. The absolute positions of the two origins are of no consequence.

Let the curve $\phi(x)$ be such that its ordinate, say z, at e' be equal, on a certain arbitrary scale, to the slope, dy/dx, of the curve $f(x)$, at the corresponding point e. Then $\phi(x)$ is the *differential curve* of $f(x)$. Conversely, $f(x)$ is an *integral curve* of $\phi(x)$. If the curve $f(x)$ is given, then by tracing it with a stylus fixed at e, the integraph is made to draw the corresponding differential curve $\phi(x)$. If $\phi(x)$ is given, then by guiding the integraph stylus e' along it, the instrument is made to draw the integral curve $f(x)$.

The differential curve gives values of the slope, or rate of change, of the given curve. The integral curve gives areas between the differential curve and its axis of abscissae. The position of the integral curve with respect to OO depends upon the point at which the area is to be equal to zero. In other words, there is a constant of integration, denoted by C. This constant must be determined by some given initial conditions, like in any problem in integration.

The best known integraph is that invented by Abdank-Abakanowicz and described in many mathematical books.[2] This device

[1] The investigation upon which this paper is based was supported by a grant from the Heckscher Foundation for the Advancement of Research, established by August Heckscher at Cornell University.

[2] See, for example, A. Galle, Mathematische Instrumente, Teubner, 1912, p. 157.

has proved to be of very limited practical use on account of its price and of delicate adjustments of parts. At the same time, there is a great need for an integraph in a number of engineering

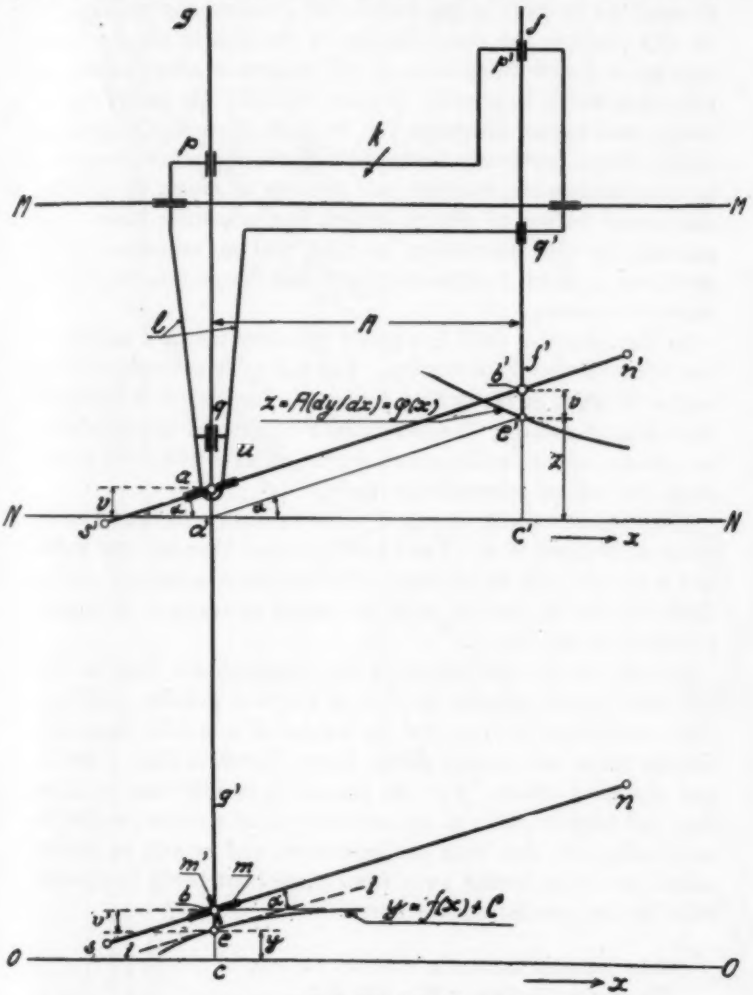

FIG. 1. *The principle of the Integraph.*

and scientific problems, involving an integration or differentiation of a given irregular curve.

The present writer was led to the development of his integraph through his interest in the hunting of synchronous machinery.[3] In this problem the determination of the size of the fly-wheel requires a double integration of the tangential-effort curve, an operation which in practice is quite tedious. He found that a simple and robust integraph can be built by using his parallel double tongs, previously described.[4] An integraph is also useful in determining the stability and flooding of ships, in plotting time-speed curves of electric trains, in computing losses in a machine by the retardation method, and in numerous other problems in which a differentiation or an integration of a given curve is necessary.

In Figs. 1 and 2, MM is a guide rail along which a carriage k can roll with very little friction. The rod gg' is movable at right angles to MM, in the guides p and q. The rod ff' is similarly guided at p' and q'. The bar sn may be rotated about point b as a center, and it carries a sharp-edged wheel m which can rotate about the axis m' perpendicular to sn.

The bracket l, firmly attached to the carriage k, has at its end a guide u, pivoted at a. The bar $s'n'$ passes through this guide and is pivoted at b' to the bar ff'. The stylus or pencil e is rigidly connected to the bar gg', and the pencil or stylus e' is rigidly attached to the bar ff'.

For the proper functioning of the integraph the bars sn and $s'n'$ must remain parallel to each in all their possible positions. This parallelism is preserved by means of a special kinematic linkage called the parallel double tongs, shown in Figs. 2 and 3, and described below. For the present it is sufficient to know that the bars sn and $s'n'$ are constrained to remain parallel to each other, but that both can be turned, and sn can be moved nearer to $s'n'$ or farther away from it, without being interferred with by the parallel double tongs.

[3] Sibley Journal of Engineering, v. *34*, No. 3:3, 1920.

[4] The American Machinist, v. *55*, p. 1050, 1921.

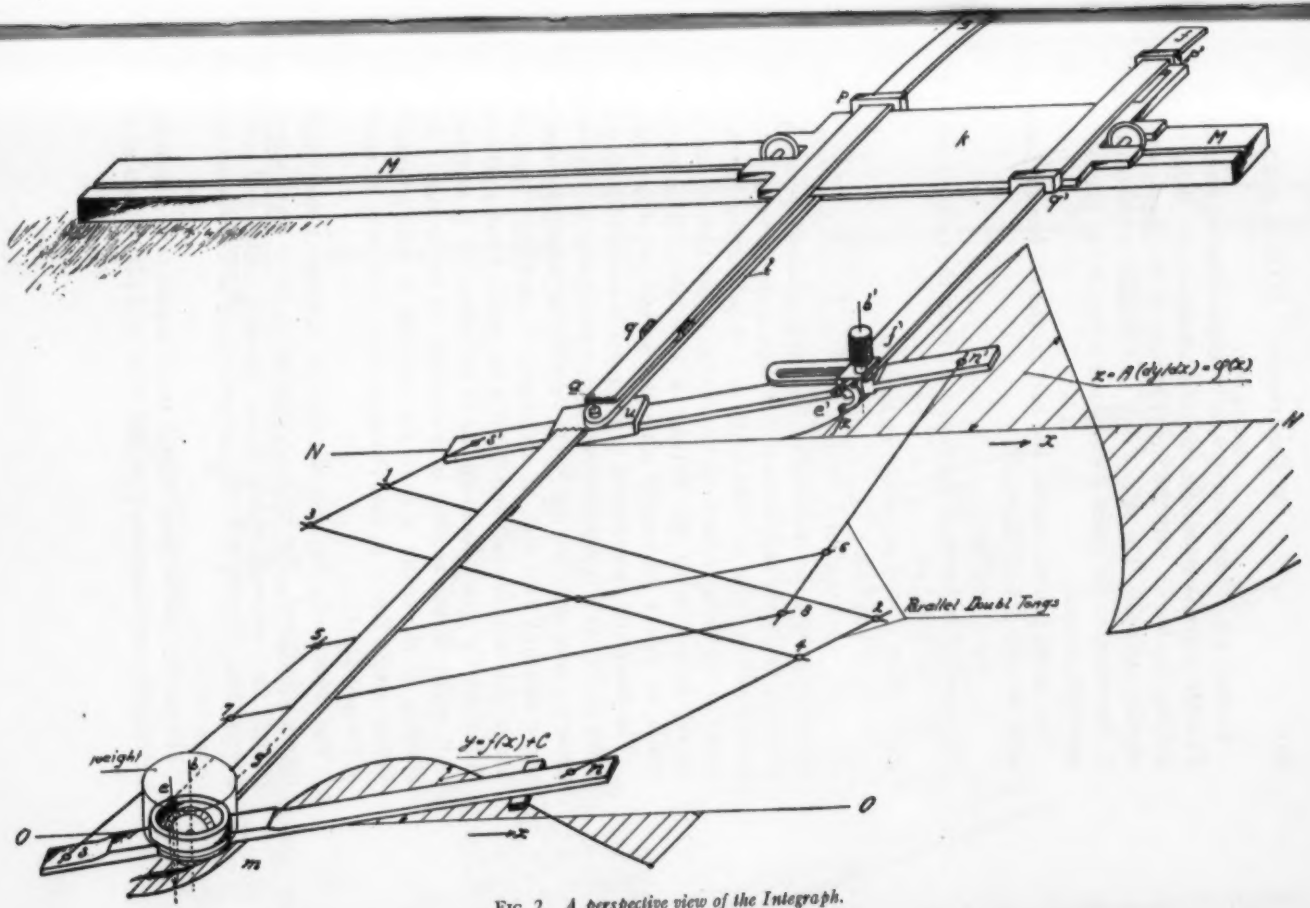

FIG. 2. *A perspective view of the Integraph.*

To draw a differential curve $\phi(x)$, corresponding to a given curve $f(x)$, the latter is traced by the stylus e, say to the right, by so guiding the bar sn that the wheel m rolls on the paper, without slipping sidewise. The axis of sn is then parallel to the tangent tt to the curve at the point e which the stylus is touching at that particular instant.

Let, in the position shown, the bars sn and $s'n'$ form an angle α with the axis NN, so that

$$\tan \alpha = dy/dx \quad \ldots\ldots (1)$$

The axis NN is drawn at the distance v from the center a, where

$$aa' = b'e' = v \quad \ldots\ldots (2)$$

Hence, from the triangle $a'e'c'$ we have

$$c'e' = a'c' \tan \alpha \quad \ldots\ldots (3)$$

or

$$z = A(dy/dx) \quad \ldots\ldots (4)$$

In other words, the ordinate of the curve $\phi(x)$, at point e', is proportional to the first derivative or slope of the curve $f(x)$ at the corresponding point e. The coefficient of proportionality is A, equal to the distance between the centers of the bars ff' and gg'. In the actual device this distance is adjustable within certain limits, so that a convenient scale can be had for either curve.

The kinematic arrangement of the parts is convertible, that is, when the stylus e' is guided along $\phi(x)$, a pencil attached at e will draw an integral curve $f(x)$. The edge of the wheel m traces a curve identical with $f(x)$, at a distance v' above it. Since the bar gg' can be moved up and down in its guides p and q, without disturbing $s'n'$, the initial ordinate of the curve $f(x)$ is arbitrary, as it ought to be, because of a constant of integration. The distance of the center a from the rail MM is also adjustable within certain limits, so that the differential curve may be raised or lowered at will.

The device can be best checked by drawing some simple curves beforehand. For example, if the given curve is an inclined straight line, $\phi(x) = Bx$, where B is a constant, its integral curve is a parabola, $f(x) = \frac{1}{2} Bx^2$. The integral curve of a sine wave is a cosine wave; etc.

The parallel double tongs (Figs. 2 and 3) consist of two identical articulated parallelograms, *1234* and *5678*, with pivot joints at all

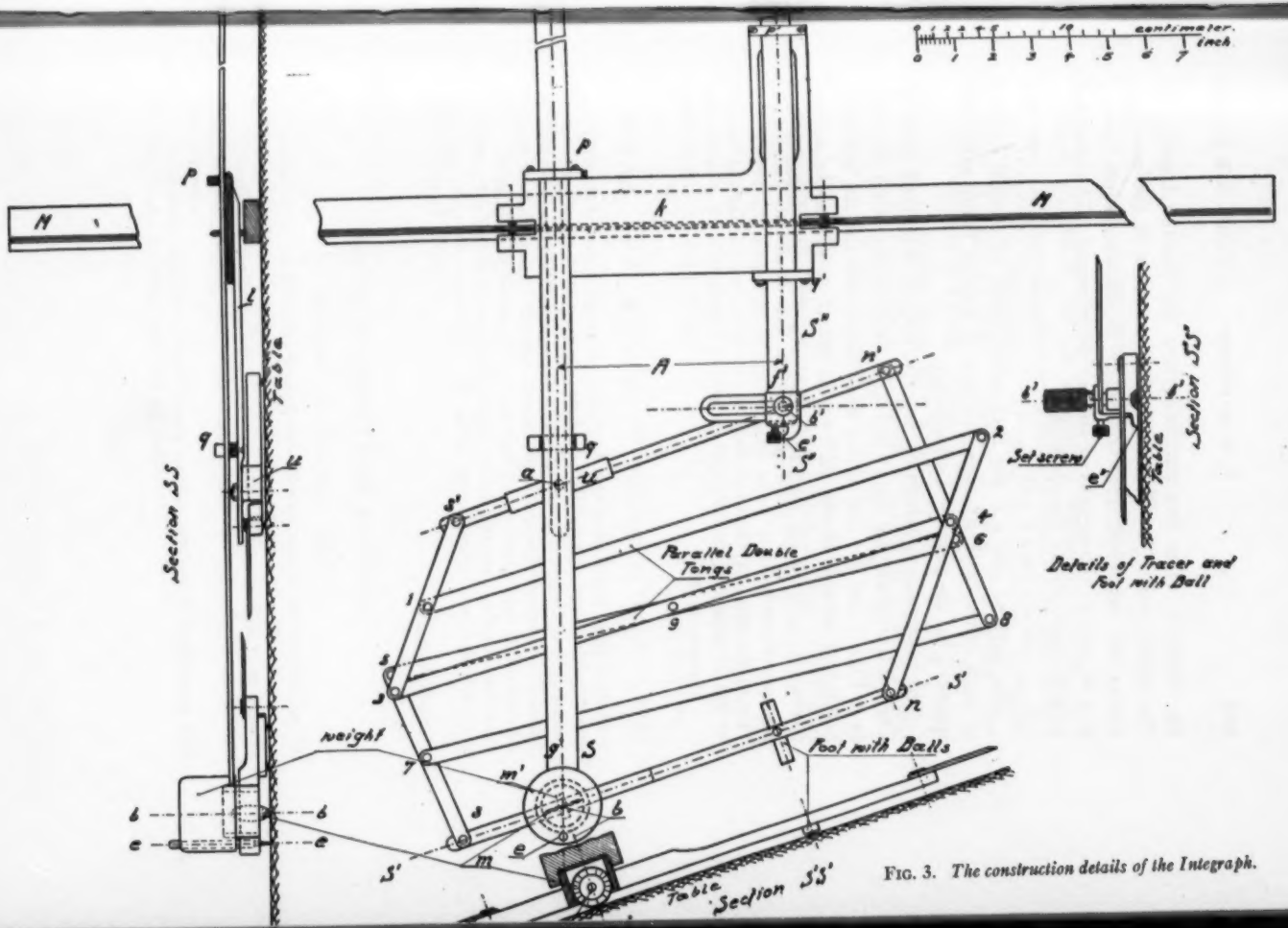

FIG. 3. *The construction details of the Integraph.*

the vertices. The two parallelograms are pivoted together at the middle point 9. The opposite short sides of the parallelograms are extended, and connected to the integraph bars at n, s, n', and s', the lengths ns and $n's'$ being equal. The extended lengths must satisfy the condition

$$n4 = s'3 = n'6 = s5 \qquad (5)$$

The bars ns and $n's'$ are then constrained to remain parallel to each other, without their position or motion being otherwise impeded by the parallel double tongs.

The construction details of the experimental integraph made in Cornell University are shown in Fig. 3, the lettering being the same as in Figs. 1 and 2. The device was constructed and the mechanical details worked out by Mr. O. K. Marti, to whom the author wishes to express his sincere appreciation for the valuable assistance rendered.

CORNELL UNIVERSITY,
ITHACA, NEW YORK.
SEPTEMBER, 1922.

REVIEWS AND NOTICES

Patent Essentials for the Executive, Engineer, Lawyer, and Inventor. A Rudimentary and Practical Treatise on the Nature of Patents, the Mechanism of their Procurement, Scientific Drafting of Patent Claims, Conduct of Cases and Special Proceedings, Including Forms. John F. Robb of the Cleveland and District of Columbia Bars with Papers by G. P. Tucker, L. W. Maxson, E. C. Reynolds, L. A. Sadler, and Edward Collins, Patent Examiners. 436 pp., Funk and Wagnalls, New York, 1922.

This book was written for the layman and presents, in nontechnical, or at least readily understood, language, the fundamental principles of patent law with which any scientist, who intends to file a patent application, should be acquainted. The book is written in a most interesting style, not at all in the dry-cut form of most treatises on legal subjects. A few hours spent in reading this work will be saved many times in the elimination of unnecessary discussion and correspondence between patent applicant and his attorney.

PAUL D. FOOTE.

Isotopes. By F. W. Aston, 152 pp., Arnold (London) 1922.

Titles of the main chapters are as follows: The Radio-active Isotopes; Positive Rays; Neon; The Mass-Spectrograph; Analysis of the Elements; The Electrical Theory of Matter; Isotopes and Atomic Numbers; The Spectra of Isotopes; The Separation of Isotopes; Various Tables. The work summarizes all data and experiments to January, 1922. Every scientist interested in the subject should have a copy of this book for reference. The book is so clearly and simply written that one who is not specializing in science will be able to read it readily and to understand the greater portion of the subjects treated.

PAUL D. FOOTE.

OPTICAL SOCIETY OF AMERICA

MINUTES OF THE EXECUTIVE COUNCIL

A meeting of the Executive Council was called to meet in Washington, Jan. 28, 1922. Present: Troland, Ives, Foote, Priest. No quorum.

A meeting of the Executive Council was called to meet in New York, Feb. 25, 1922. Present: Troland, Ives, Lomb, Gale, Southall. No quorum. (Southall Secretary, protem.)

An informal conference of the following members was held in Washington, April 22, 1922: Foote, Forsythe, Gale, Merritt, Priest.

As a result of these conferences and correspondence, President Troland on May 4, 1922, declared the following resolutions adopted by unanimous vote of the Council:

To amend Article II, Section 2 of the By-Laws to read as follows: "No officer or member of the Society except the editor-in-chief and the assistant editor-in-chief and business manager of the Journal shall receive any remuneration for his services."

To Amend Article V of the By-Laws by inserting after editor-in-chief the words "and the assistant editor-in-chief and business manager."

RESOLVED that a ballot be mailed to the members of the Society to vote upon an amendment to the constitution to read as follows: "The assistant editor-in-chief and the business manager of the Journal shall be ex officio a member of the Executive Council."

RESOLVED that Mr. Adolph Lomb, as treasurer of the Optical Society, is hereby designated by the Executive Council to sign the agreement between the Optical Society of America and the Association of Scientific Apparatus Makers relating to the financing and publication of the JOURNAL OF THE OPTICAL SOCIETY OF AMERICA AND REVIEW OF SCIENTIFIC INSTRUMENTS.

RESOLVED that the agreement between the Optical Society of America and the Association of Scientific Apparatus Makers relating to the publication of a Journal to be known as the JOURNAL OF THE OPTICAL SOCIETY OF AMERICA AND REVIEW OF SCIENTIFIC INSTRUMENTS for a period of two years beginning April 1, 1922, when signed by the authorized representatives of the said Society and Association, be hereby ratified confirmed and approved by the Executive Council acting for the Optical Society of America.

RESOLVED that the financial transactions of the Optical Society of America and of the JOURNAL OF THE OPTICAL SOCIETY OF AMERICA AND REVIEW OF SCIENTIFIC INSTRUMENTS shall hereafter be separately managed, those of the Society remaining in the hands of the treasurer whereas those of the Journal shall be entirely under the control of the business manager of the Journal, that the business manager of the Journal shall be responsible for all collections and disbursements involved in the publication of the Journal and that contributions or payments made by the Society towards the expenses of the Journal should be made by the treasurer to the business manager of the Journal in a manner mutually agreed upon by said officers.

RESOLVED that the disbursement of the contribution of $2,500 made by the National Research Council towards the support of the JOURNAL OF THE OPTICAL SOCIETY OF AMERICA AND THE REVIEW OF SCIENTIFIC INSTRUMENTS shall be entirely in the hands of the business manager of the Journal, who shall be authorized to sign all vouchers and requisitions connected with this appropriation.

RESOLVED that the business manager of the Journal shall make an annual report as of April 1st of each year and that this report shall be duly audited and published in the Journal.

RESOLVED that the committee on membership be authorized to drop from membership the names of such persons as are in arrears in accordance with Article I, Section 5, of the By-Laws.

RESOLVED that the business manager of the Journal be directed and authorized to revise the contract with the George Banta Publishing Company of Menasha, Wisconsin, and to submit the same to the president and the editor-in-chief for approval, and that after receiving such

approval the business manager be empowered to sign the contract on behalf of the Optical Society.

RESOLVED that the printed reports of the sub-committees of the Optical Society committee on nomenclature and standards shall be advertised in the Journal as for sale at a price per copy depending upon the number of pages, the number of such reprinted reports and price to be determined by the publication committee, and the proceeds of such sales to go to the treasury of the Optical Society.

IRWIN G. PRIEST,
Secretary.

WASHINGTON, D. C.
AUG. 14, 1922.

OPTICAL SOCIETY OF AMERICA

NEW MEMBERS

The following new members have been duly elected by the Executive Council:

REGULAR

No. 325. William L. Benedict, Mayo Clinic, Rochester, Minn.
No. 326. George Walter Stewart, Hall of Physics, Iowa City, Iowa.
No. 327. Arthur Edward Ruark, 6010 Henderson Avenue, Govans, Baltimore, Md.
No. 328. Francis G. Pease, Mount Wilson Observatory, Pasadena, Calif.
No. 329. Frank Walter Weymouth, Stanford University, California.

ASSOCIATE

No. 330. Ting Supoa, 38 M. D. Hall, University of Chicago, Chicago, Ill.

BY TRANSFER FROM ASSOCIATE TO REGULAR

Reinhard A. Wetzel has been transferred to Regular Membership.

David Rines, who was elected to Associate Membership at Rochester, October 24, 1921, through lack of complete information as to his qualifications, has been made a Regular Member.

IRWIN G. PRIEST,
Secretary.

August 31, 1922.

NOTICES

OPTICAL SOCIETY OF AMERICA

TELLERS' REPORT

To Irwin G. Priest, Secretary:

We, the undersigned regular members of the Optical Society of America, have counted the ballots cast on the following proposed amendment to the Constitution:

"The assistant editor-in-chief and business manager of the JOURNAL shall be ex-officio a member of the Executive Council."

We have verified our count and certify the following to be the true result:

In favor of the amentment.....95
Opposed to the Amendment..... 0

(Signed) K. S. GIBSON
M. K. FREHAFER
Tellers.

September 2, 1922.

In accord with the above report and the Constitution's provision for its amendment, the above amendment is hereby declared adopted.

(Signed) LEONARD T. TROLAND
President
IRWIN G. PRIEST
Secretary

September 5, 1922.